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Beyond Subitizing: Symbolic Manipulations of Numbers[1]

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Abstract. This study tested the hypothesis that subitizing ability may cause achievement differences in mathematics especially for students with mathematics learning disabilities. Students from 1st through 4th grade were applied to curriculum based math achievement tests (MAT). Based on MAT scores, they were divided into four groups as Mathematics Learning Disorder (MLD) risks, low achievers (LA), typical achievers (TA), and high achievers (HA). All students were asked randomly and canonically arranged dot enumeration tasks with 3 through 9 dots. Median response times (MRT) were calculated for each task and plotted for each grade level and task types. There were virtually no differences in MRTs for number 3 and 4. On the other hand, the MLD risk group spent relatively more time on enumerating canonically arranged dots from 5 through 9. Results provided more support for the claim that rather than subitizing, numerosity coding mechanisms or the type of symbolic quantity manipulations is different in children with different mathematical achievements especially the lower group, the MLD risk group.

Keywords: subitizing, numerosity coding, math achievement, symbolic manipulations of quantities

1 An earlier version of this paper was presented at the 2014 Meeting of the Special Interest Group (SIG) 22 "Neuroscience and Education" organized by the European Association for Research on Learning and Instruction (EARLI) on 12th to 14th June 2014 in Göttingen, Germany.

1. Introduction

Many researchers (Desoete, Ceulemans, Roeyers, & Huylebroeck, 2009; Landerl, Bevan, & Butterworth, 2004; Landerl & Kolle, 2009) claimed that deficits in subitizing mechanism, responsible for quickly enumerating small number of objects at a glance, may cause severe learning disabilities in mathematics. Butterworth (2010) on the other hand proposed an alternative hypothesis in that he claimed the deficit lies in an inherited system for sets of objects and operations on them (numerosity coding) on which arithmetic is built. However, there is a few empirical evidence to support both of the claims strongly. They are either single case studies or position papers. The purpose of this study was to test the hypothesis that subitizing may cause achievement differences in mathematics especially for students with mathematics learning disabilities.

1.1 Theoretical background

Human cognition has at least 5 core systems for the foundations of human knowledge, one of which deals with numbers (Spelke & Kinzler, 2007). Human number system and possibly some other species are thought to have two separate sub systems to deal with different aspects of number (Feigenson, Dehaene, & Spelke, 2004). For example Lemer, Dehaene, Spelke, and Cohen (2003) suggested that numerical abilities rest on the integration of two distinct systems, a verbal system and a non-symbolic system. Verbal system represents the numerical magnitude as exact quantities while the non-symbolic or analog system represents approximate quantities.

The approximate number system (ANS) deals with large numbers (>4) in an approximate fashion. As small as 6 months old infants can discriminate 8 from 16 represented in visual (Xue & Spelke, 2000) or auditory sequences (Lipton & Spelke, 2003) but not 8 from 12. On the other hand 9-month-old infants can discriminate both but not 8 from 10 (Lipton & Spelke, 2003). There is also evidence that 9-month-old infants successfully add and subtract over numbers of items that exceed object-tracking limits (McCrink & Wynn, 2004), that is out of subitizing range (>4). However, in both numerosity comparison, and addition and subtraction situations, there is a set size signature, called Weber fraction, which develops from infancy through adulthood. This basic capacity seems to develop prior to language and symbolic counting and still to be in use in adulthood.

The other subsystem is called exact (or small) number system (ENS). This basic number processing ability is also intact starting from infancy. Even 5-days-old infants can discriminate 2 dots from 3 dots by means of subitizing but not 4 dots from 6 dots (Antell & Keating, 1983). Six-month-olds can control small numbers of objects even under addition and subtraction operations (Wynn, 1992). Adults can also use this system to quickly enumerate small numbers of objects up to four. If the number of objects is more than four and if there is time available then counting or calculation operations are used within this system. However, a different system is engaged to enumerate the objects under limited time constraints.

Some researchers (Butterworth, 2003; S. Dehaene, 1997) believe that any disorder or malfunction in either of these systems can cause specific learning disabilities in mathematics. In fact, there are several hypothesis about the root causes of very low achievement in math or dyscalculia. One posits that the deficit in approximate number system (ANS) causes mathematics learning difficulties. Others claim that dyscalculia stems largely from the deficit in exact number system (ENS) or subitizing, quickly enumerating small numbers of objects usually less then 5. Still others state that dyscalculia is caused by the deficit in accessing magnitudes from symbols or vice versa, called access deficit hypothesis (ADH). Rousselle and Noel (2007) for instance claimed that children with mathematics learning disabilities have difficulty in accessing numerical magnitude from symbols rather than in processing numerosity per se. Butterworth (2010) on the other hand claimed that a deficit in numerosity coding, not in the approximate number system or the small number system, is responsible for dyscalculia. In contrast to this claim Landerl and Kolle (2009) did not find much evidence that dyscalculic children process numbers qualitatively differently from children with typical arithmetic development.

Available evidence indicates that a fine grained research is needed to further clarify these issues. The purpose of this study was then to test the hypothesis that either subitizing or numerosity coding mechanisms is different in children with different mathematical achievements by using dot counting paradigms.

2. Methods

2.1 Participants

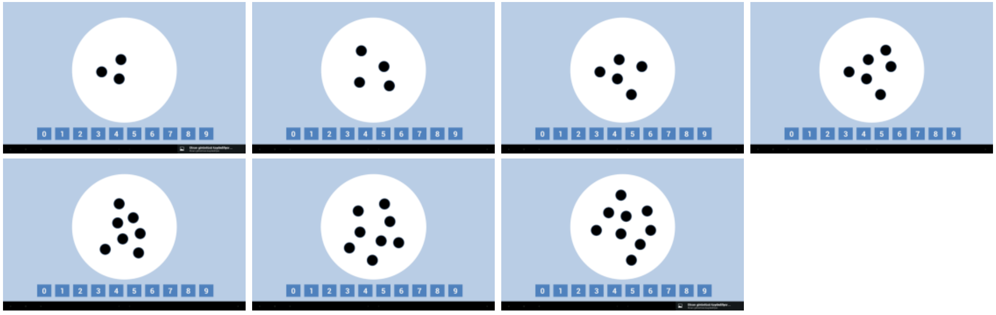
Participants were randomly selected from 12 elementary schools, located four different SES locations with an intention to draw a representative sample of students from 1st to 4th grade within a metropolitan area in a mid-Anatolian city. Initially, data were collected from 487 students. Six students were excluded from the sample since they were diagnosed with some sort of general learning disorders and/or mainstreamed in regular classrooms. The final sample consist of 125, 126, 121, and 109 participants (481 in total) from 1st, 2nd, 3rd, and 4th grade respectively. There are approximately equal numbers of boys and girls in each grade level and in total. We did not control for IQ for the practical difficulties, so this is a limitation.

2.2 Data collection tools

There were mainly three tests for data collection. First, all participants were administered a math achievement test (MAT) developed by Fidan (2013) based on the number domain of the Turkish State Curriculum (MEB, 2004). There were different achievement tests for each grade level. The tests contained 13, 15, 16, and 24 items for the first, second, third, and fourth grade respectively. The math achievement test is an untimed test but the administration took one class hour (approximately 40 minutes) for the students.

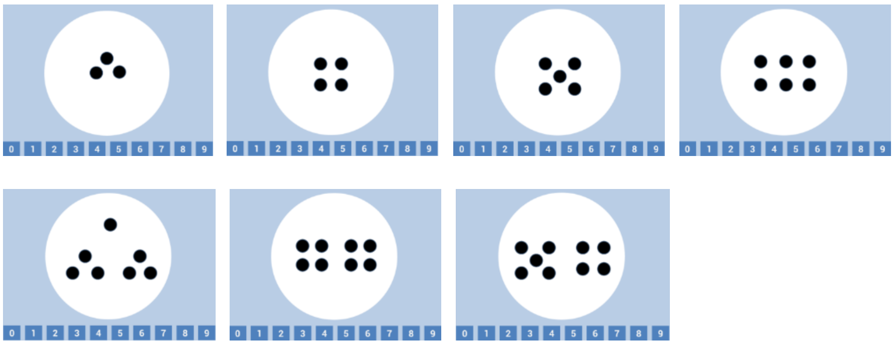
The second test contained dot counting tasks (RDC). The dots were randomly arranged in each task from the numerosity 3 to 9 (see Figure 1 for details). The test contained 14 tasks representing each numerosity twice.

Figure 1. Random dot counting tasks



The third test was, in many respects similar to the second test except that the dots were arranged canonically or domino like patterns (CDC) (see Figure 2 for details). Canonic dot patterns are considered to be symbolically manipulated (Piazza, Mechelli, Butterworth, & Price, 2002). So we expected that randomly arranged dots are enumerated differently from canonically arranged dots by students with different achievement levels.

Figure 2. Canonic dot counting tasks



2.3 Procedure and Analysis

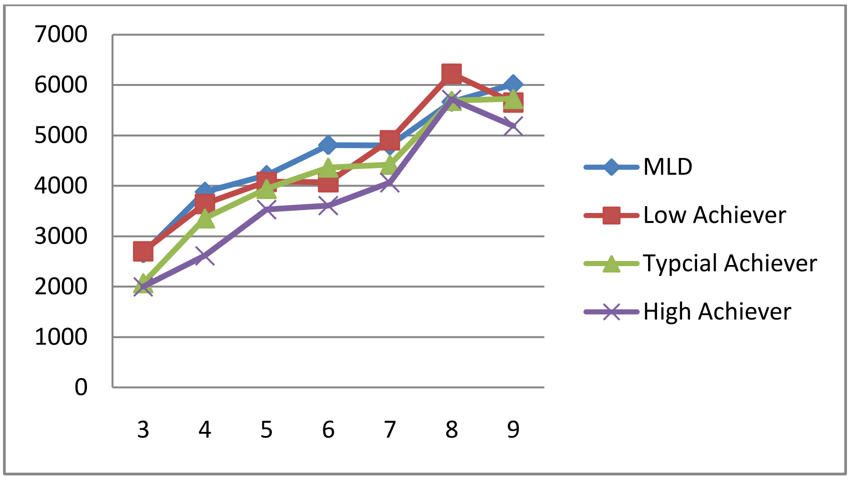
Raw scores were gathered from the math achievement tests. Students were placed in four achievement groups based on their math achievement scores, in each grade level. Earlier research (Barbaresi, Katusic, Colligan, Weaver, & Jacobsen, 2005; Shalev, Manor, & Gross-Tsur, 2005) reported the prevalence of dyscalculia or MLD roughly from 5% to 14%. In average we placed the lowest 10% of the students in MLD risk group, 11-25% in low achievement, 26-95% in typical achievement and >95% in high achievement group. Finally, we consulted teachers for their opinions about the students to make sure that students' math scores reflected their general situations in mathematics. Students with some other learning difficulties were also excluded. Defining the MLD risk group was loose in this study so this was another limitation of the current study.

We calculated groups' median response times (MRT) for CDC and RDC tests. Based on these scores and achievement groupings we plotted the data to see the changes in response times from the number 3 to 9. RDC and CDC data were plotted separately for each grade level. Each achievement group has a separate line in the graphs.

3. Results

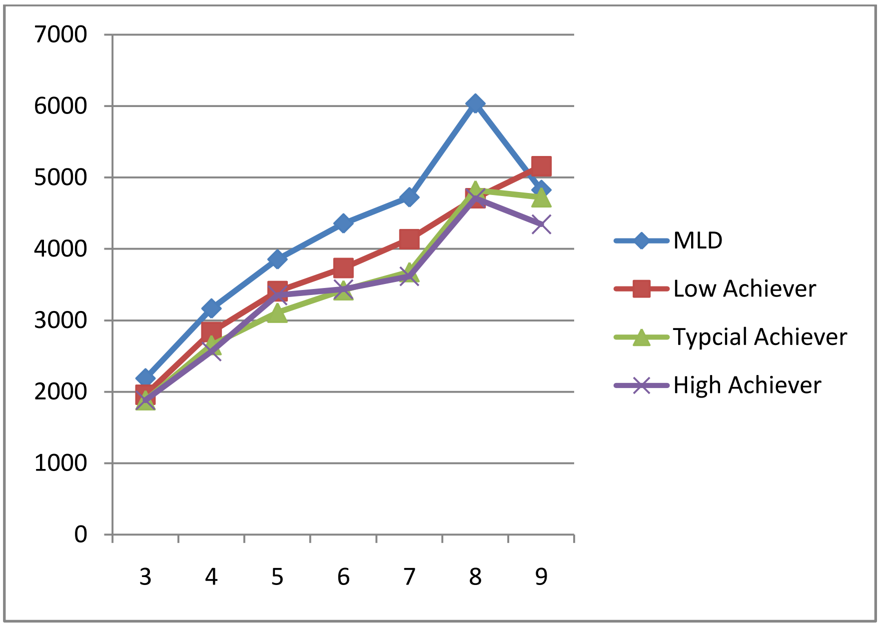
As seen in Figure1, first graders had a steadly increasing median response times (MRT) for enumarating the numbers from 3 to 9. Students in all achievement groups had the similar patterns of MRTs except that the lower groups had relatively higher MRTs.

Figure 1. Median response times of 1st graders to RDC tasks



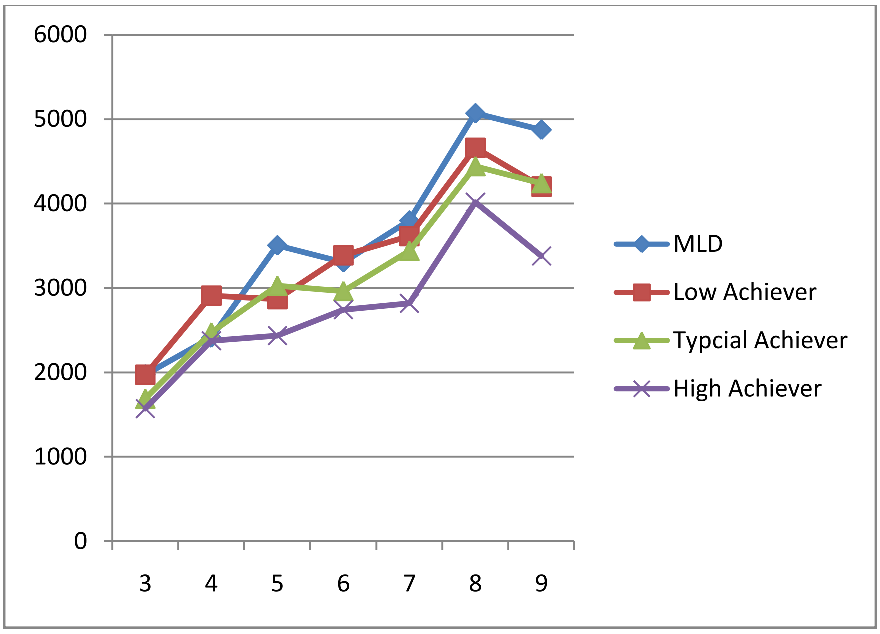
Second graders had also a gradually and steadly increasing median response times for enumerating the numbers from 3 to 9. As seen in Figure 2, the MLD group had relatively higher median response times than the other groups. The three other groups have almost similar median response times. The discrepancy in MLD group's MRTs is getting more and more while the number of dots increases from 3 to 8 except nine.

Figure 2. Median response times of 2nd graders to RDC tasks



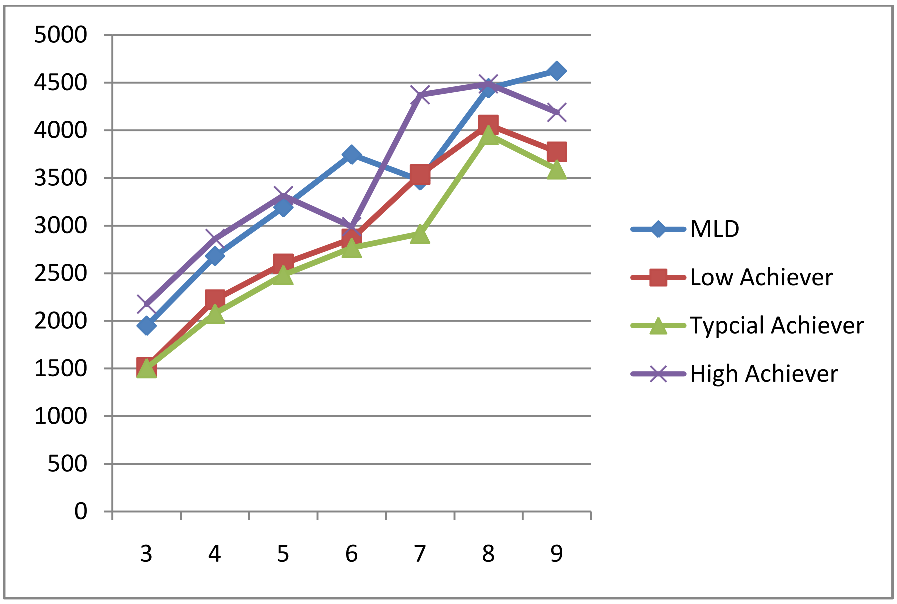
Third graders'patterns of MRT is also similar to the first and second graders in terms of steady increase. This time however, the achievement groups especially the lower groups are closer to each other.

Figure 3. Median response times of 3rd graders to RDC tasks



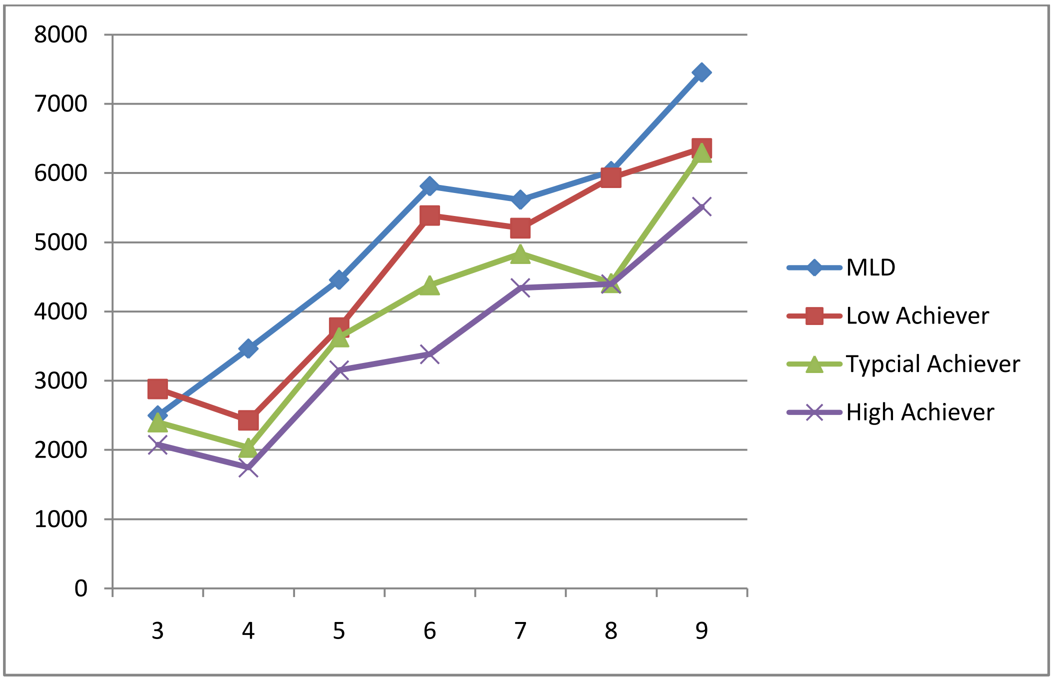
Fourth graders' data showed a suprizing and unexpected pattern in terms of high achievers' MRT. Except the number 6, high achievers got higher MRTs from other groups. However, all other patterns are similar to the first, second, and third graders'.

Figure 4. Median response times of 4th graders to RDC tasks



The graphs obtained from the canonic dot counting (CDC) test are more illustrative than RCD tests in that CDC tasks seem to be more discriminative than RDC tasks. As seen in Figure 5, 6, 7 and 8, except for third graders, in all other grades MLD group showed an MRT pattern visibly different from other achievement groups especially for higher numbers.

Figure 5. Median response times of 1st graders to CDC tasks



Additionally, except for the first graders' MRTs for the number 4, there is virtually no difference between the achievement groups in terms of MRTs for the number in the subitizing range (number 3 and 4). In other words, the MRTs are very close to each other both for achievement groups and numbers (ie. 3 and 4). That means the tasks of enumerating canonically arranged 3 and 4 is the same for all students including very low achievers. This is the most striking finding of this study.

From first to fourth grade, on the other hand while the other achievement groups are getting closer to each other in terms of MRTs the MLD risk group is getting hiher and higher MRTs from the number 5 though 9. This is the second most important finding of this study.

Figure 6. Median response times of 2nd graders to CDC tasks

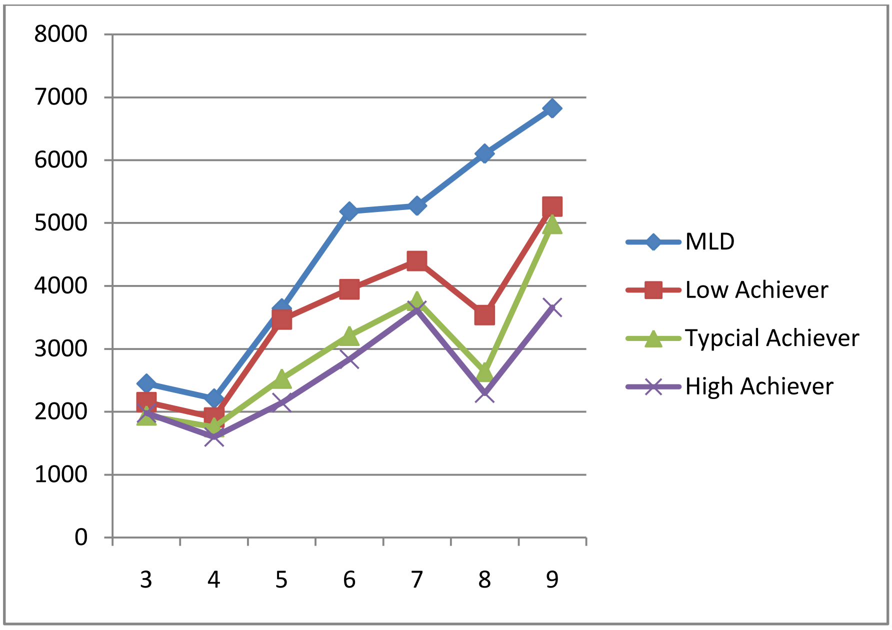


Figure 7. Median response times of 3rd graders to CDC tasks

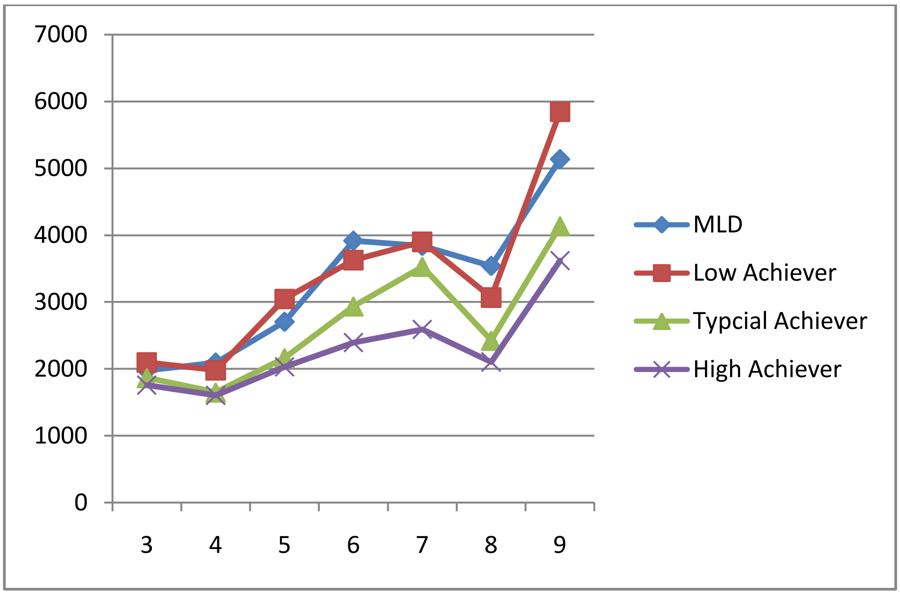
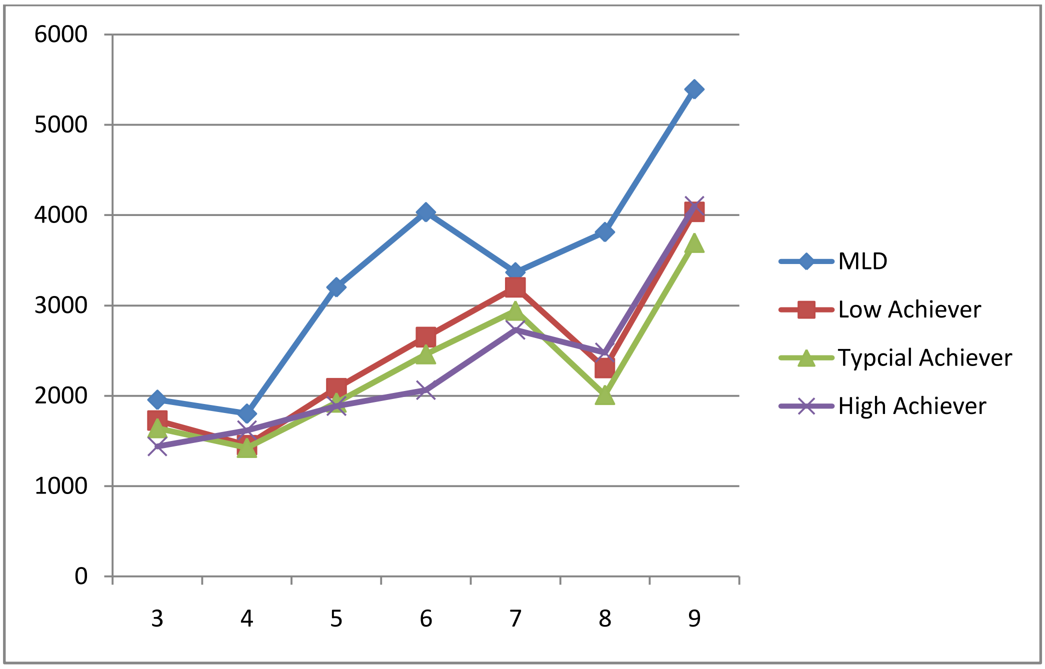


Figure 8. Median response times of 4th graders to CDC tasks



4. Discussion

The findings of this study showed that there might be differences among the achievement groups in terms of subitizing ability. This seems less likely however. Except for the first graders, there were virtually no differences among achievement groups in terms of enumerating the numbers in the subitizing range (3 and 4) if the dots were arranged canonically. Only the MLD risk group in the firs grade spent longer time for enumerating 4 than they did for 3 dots. They all spent almost equal amount of time to determine the number of dots shown. If the dots were randomly arranged all groups behaved again similarly. However, they spent relatively longer time to enumerate 4 dots than they did for 3 dots. That is they all used similar inferior strategies.

When the dots were arranged in canonic, domino like patterns students treated 3 dots and 4 dots as if they are almost the same. They all spent virtually the same time for enumerating each number of dots. After 4 dots however, the MLD group spent relatively more time for enumerating the number of dots. It seemed that, they used still inferior strategies like counting on after subitizing one of the groups while normally achieving students were applying simple calculations on the separately subitized groups of dots. Imposing groupings onto randomly arranged dot sets seemed not possible for all groups. Therefore, we obtained virtually no differences among the groups in terms of MRTs for all numbers.

Taken together, it seems that the students have almost similar subitizing systems for small numbers but while normal and high achieving students were making further symbolic manipulations on subitized groups of dots very low achievers were still using inferior strategies. We believe with Butterworth (2010) that a deficit in numerosity coding is responsible for mathematics disorder. Further research are needed, however to strongly claim that numerosity coding rather than subitizing mechanisms are different in MLD and normally achieving students. Carefully designed behavioral and brain imaging research can highlight this problematic issue. Experimental studies with instructional interventions may also help clarify. For example, instructing students to make groupings might differentiate students with MLD more precisely from normally achieving peers. Similarly Iuculano, Tang, Hall, and Butterworth (2008) claimed that low numeracy has not to do with a poor grasp of exact numerosities (ie. subitizing), but more related to inefficient use of symbolic numerals.

In a brain imaging study, Piazza et al. (2002) gave adults enumeration task on visual arrays of dots that varied in numerosity (1–4 and 6–9 dots) and spatial arrangement (canonical and random) to directly compare subitizing and counting. They showed that counting, relative to subitizing, was correlated with increased activity in the occipitoparietal network, while subitizing did not show areas of increased activation with respect to counting. Surprisingly, they concluded that results speak against the idea of the two processes being implemented in separable neural systems.

This research showed that subitizing is necessary but not sufficient for doing further arithmetic beyond simple counting. Further numerical manipulations are needed to do faster arithmetic. In fact many animal spicies can subitize but cannot do arithmetic beyond a simpler form (Stanislas Dehaene, Dehaene-Lambertz, & Cohen, 1998). This research also clarified that these further manipulations were actualized via symbolic manipulations since we know that canonicly arranged dot patterns are thought to be symbolically manipulated (Piazza et al., 2002). Group differences were more pronounced in enumerating canonic dot patterns. An educational intervantional study will shed more light in this issue. Abstracting and symbolizing are two important processes in the course of arithmetic development.

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