International Journal of Learning, Teaching and Educational Research
Vol. 10, No. 1, pp. 120-141, January 2015

Building the process of transformation teaching and learning according to the constructivism with the help of dynamic geometry software in Vietnam

Ngoc-Giang Nguyen
Doctor's degree student of The Vietnam Institute of Educational Sciences,
101 Tran Hung Dao, Hanoi, Vietnam

Anh-Tai Phan
Doctor of Sai Gon University
273 An Duong Vuong Street, Ho Chi Minh city, Vietnam

Abstract. The theory of constructivism is generally attributed to by Jean Piaget. Two important conceptions of Piaget in the constructivism are *assimilation* and *accommodation*. *Assimilation* is the process in which we get new knowledge similar to the existing knowledge, and then this new knowledge can be directly incorporated in an already existing framework. In other words, students can deal with the new situation based on the existing knowledge. *Accommodation* is the process in which we get new knowledge which is different from an existing framework, and then this framework can be changed to fit the new knowledge. Applying dynamic geometry software is a new researched way with many good results. Dynamic geometry software helps us learn mathematical concepts, theorems and solve problems. In addition, dynamic geometry software also provides some informatics skills for the user.

Keywords: Constructivism, dynamic geometry software, process.

1. Introduction

Applying dynamic geometry software makes the learning process better and more effective. Learners use the software in predicting, verifying as well as developing their knowledge. The expansion of applying this software to various levels and forms depends on specific conditions of the problem.

Constructivism is now being mentioned in a lot of articles (e.g. [2], [5], [6], [7]). In this paper, we research the applying of constructivism to transformation learning with the help of dynamic geometry software in Vietnam. The paper gives processes as well as examples of learning concepts, theorems and solves problems according to constructivism with the help of dynamic geometry software. The strong point of constructivist learning is that it helps to develop dependent thoughts, creative thoughts and scientific passion. Students can learn lessons deeply and remember them for a long time. However, its weak point is that it takes teachers a lot of time and effort to write lessons according to constructivism. This article also performs the survey of applying informatics to constructivist learning at upper secondary schools in Vietnam.

2. Learning transformations with the help of dynamic geometry software according to constructivism

2.1. The process of using dynamic geometry software in learning transformation according to constructivism

In learning mathematics at upper secondary schools in Vietnam, the understanding of mathematical concepts is an important factor. Mathematical conceptions are only formed deeply if students join in the process of building them actively and initiatively. We give the process of using dynamic geometry software in learning conceptions according to constructivism as follows :

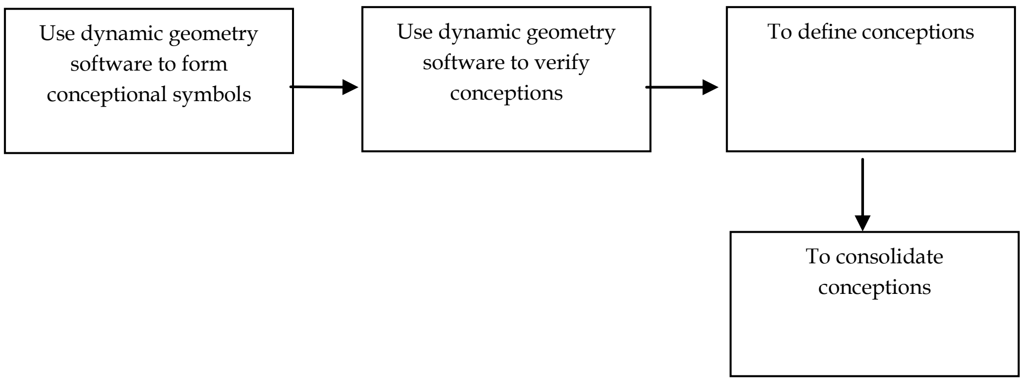


Figure 1. Learning transformation according to constructivism with the help of dynamic geometry software

2.2. An example illustrating the transformation concept according to constructivism with the help of dynamic geometry software

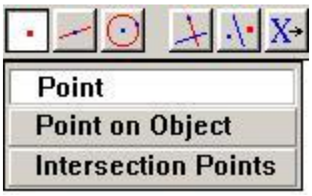
In order to form transformation concept, we use dynamic geometry software such as Cabri, GeoGebra, Sketchpad, … to draw dynamic figures. After students observe examples, ask questions and give remarks, they themselves conclude the definition of the conception.

The following are some tools in the Cabri and GeoGebra software:

a) Some tools in the Cabri software

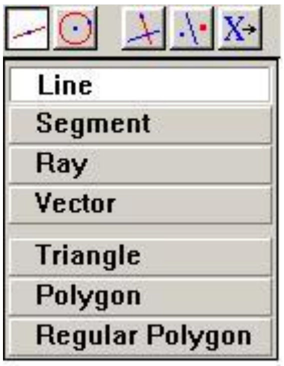
1. Point

Use the tool $\boxed{\text{Point}}$ to draw a point A.



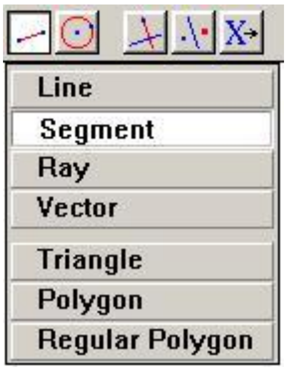
2. Line

Use the tool Line to draw a line a.



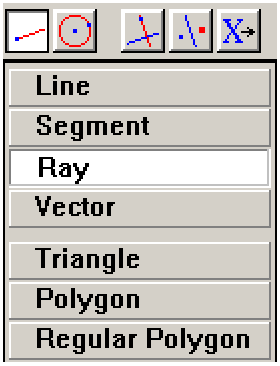
3. Segment

Use the tool Segment to draw a segment AB.



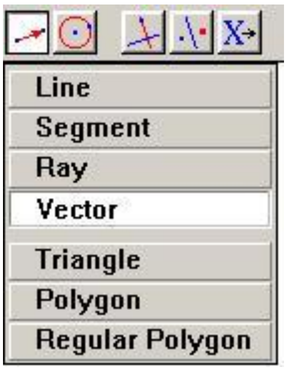
4. Ray

Use the tool Ray to draw a ray from A to B.



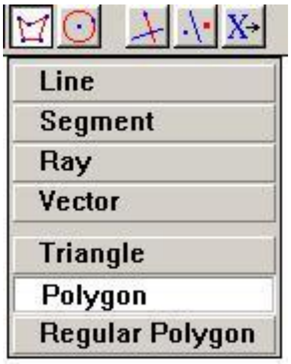
5. Vector

Use the tool Vector to draw a vector from A to B.



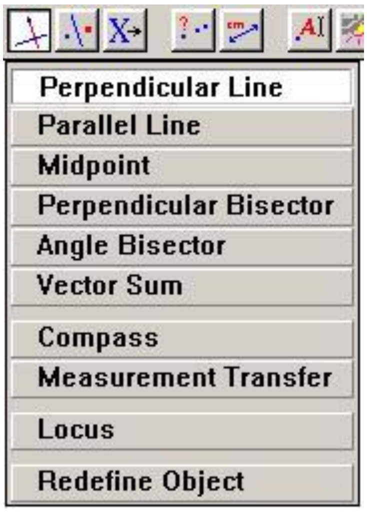
6. Polygon

Use the tool Polygon to draw a polygon.



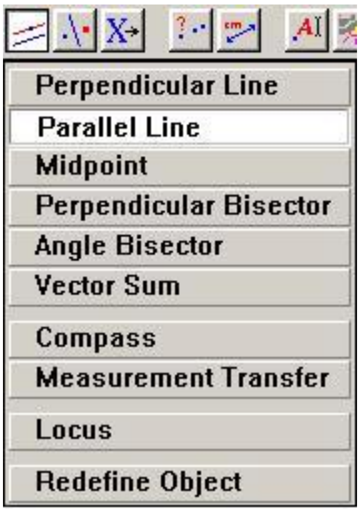
7. Perpendicular line

Use the tool Perpendicular Line to draw a perpendicular line d to a.



8. Parallel Line

Given a line AB. Draw a line b passing through a given point C, which is parallel to AB : Parallel Line $\to C \to AB$.

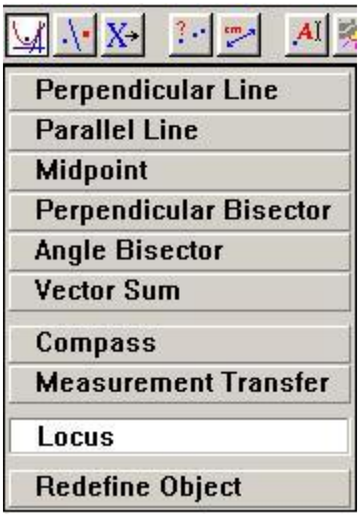


9. Circle

Use the tool Circle to draw a circle with center O and radius r.

10. Locus

Use the tool Locus to find the locus of a movable object.



11. Trace

Use the tool Trace On/Off to make (cancel) the trace of an object.

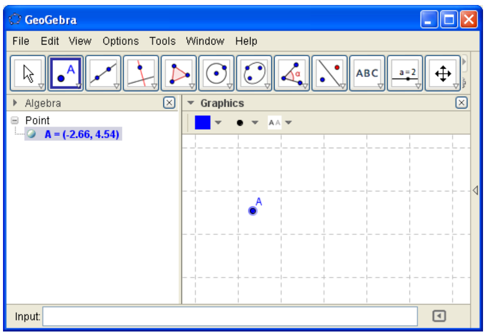
12. Animation

Use the tool Animation to move an object.

b) Some tools in the GeoGebra software

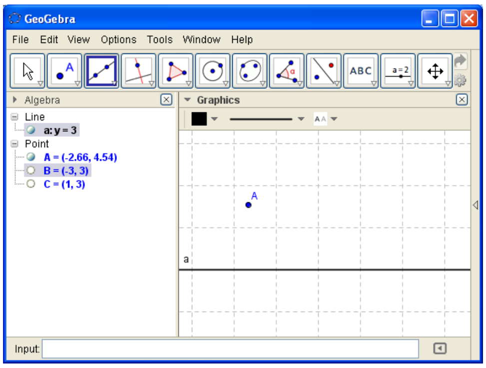
1. Point

Use the tool Point to draw a point A.



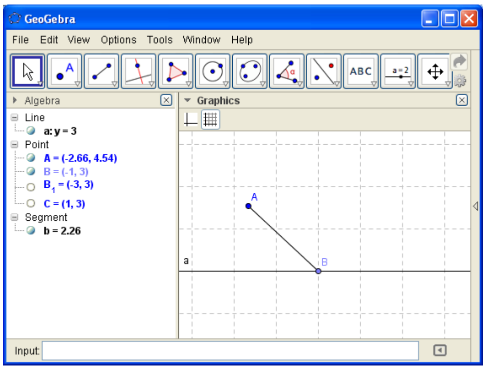
2. Line

Use the tool Line to draw a line a.



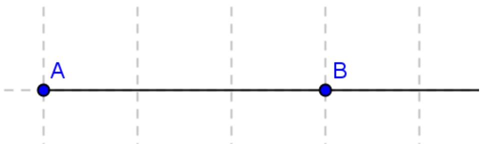
3. Segment

Use the tool Segment to draw a segment AB.



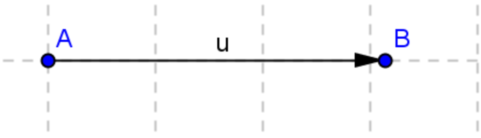
4. Ray

Use the tool Ray to draw a ray from A to B.



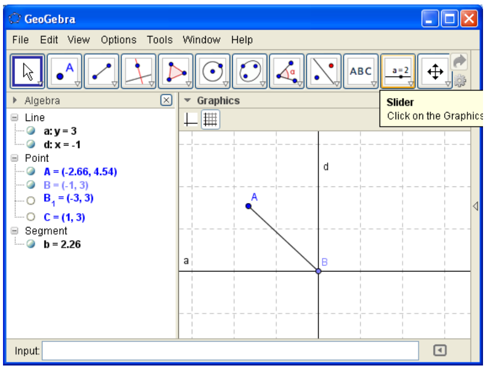
5. Vector

Use the tool Vector to draw a vector from A to B.



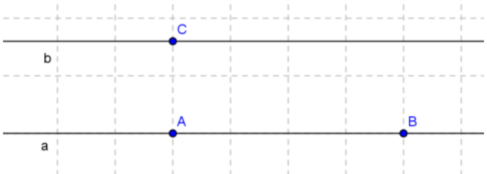
6. Perpendicular Line

Use the tool Perpendicular Line to draw a perpendicular line d to a.



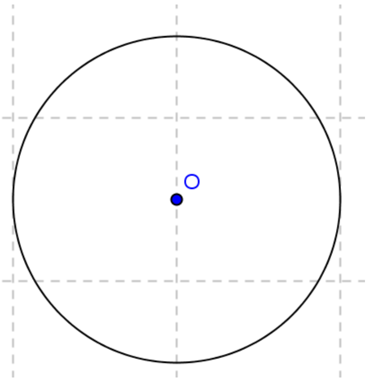
7. Parallel Line

Given a line AB. Draw a line b passing through a given point C, which is parallel to AB : $\boxed{\text{Parallel Line}} \rightarrow C \rightarrow AB$.



8. Circle with Center Through Point

Use the tool $\boxed{\text{Circle with Center Through Point}}$ to draw a circle with center O and radius r.



9. Locus

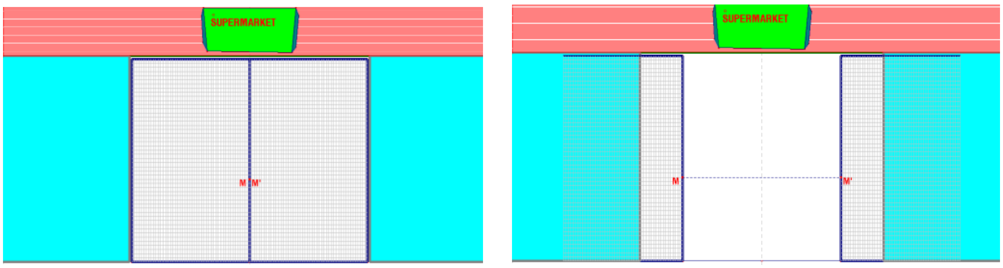
Use the tool $\boxed{\text{Locus}}$ to find the locus of a movable object.

Example 1

Observe the two entrance doors of a supermarket, and give remarks on two positional points M, M' compared with the midline of the entrance.

Using dynamic geometry software to draw :

a) b)



Figures 2a, b

Remarks

Two points M, M' are symmetric with respect to the midline of entrance door.

Example 2

Two given lines a and d satisfying that they intersect at A. With each of points M on d, draw point M' symmetric to the point M with respect to a. When M moves on d, give remarks on the positional points M'.

Using dynamic geometry software to draw :

- Draw line a.
- Draw line d meeting a at A.
- Draw point M on d.
- Draw point M' symmetric to point M with respect to a.
- Dragging point M on d gives us point M' such that it makes "traces" forming line d'.

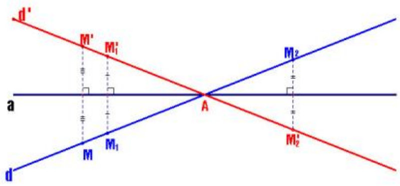


Figure 3

Remarks

M' moves on d through A such that d' and d take a as a bisector line of a pair of vertically opposite angles formed by d' and d.

Example 3

Having two given parallel lines a and d. With each of points M on d, draw point M' symmetric to point M with respect to a. When M moves on d, give remarks on the positional points M'.

Using dynamic geometry software to draw :

- Draw line a.
- Draw line d // a.
- Let point M be on d.
- Draw point M' symmetric to point M with respect to a.
- Dragging point M on d gives us point M' so that it makes "traces" forming line d' // a.

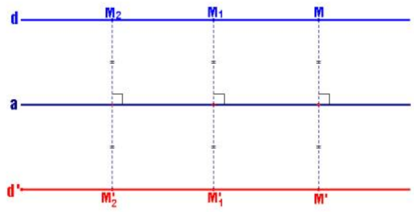


Figure 4

Remarks
M' moves on line d' such that d' and d are parallel and equidistant from a.

Example 4
Given line a and circle (O). With each of points M on (O), draw point M' symmetric to point M with respect to a. When M moves on (O), give remarks on the positional points M'.
Use dynamic geometry software:
- Draw circle (O).
- Draw line a and circle (O).
- Let M be a point on (O).
- Draw point M' symmetric to point M with respect to a.
- Dragging point M on (O) gives us point M' such that it makes "traces" forming circle (O').

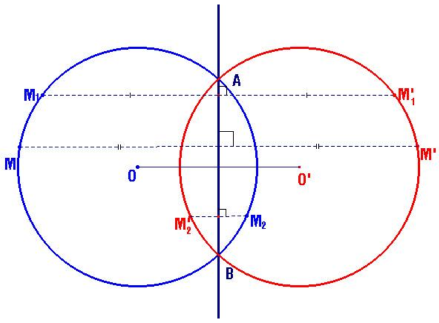


Figure 5

Remarks
M' moves on circle (O') being equal to (O).

Example 5
Given line a and a figure (H) (for example, the figure of pine tree). With each of points M on (H), draw point M' symmetric to point M with respect to a. When M moves on (H), give remarks on the positional points M'.
Using dynamic geometry software to draw :
- Draw line a.
- Draw figure (H).
- Let M be a point on (H).
- Draw point M' symmetric to point M with respect to a.
- Dragging point M on (H) gives us point M' such that it makes "traces" forming figure (H).

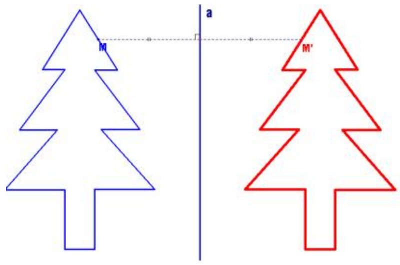


Figure 6

Remarks

We see that when M moves on (H), M' makes "traces" forming figure (H') so that (H') is similar to (H). We also realize that if we fold "the plane containing the paper" based on line a such that two parts of this paper are close, (H) is coincident with (H').

From the given examples, and observing the above figures, we realize that they have a common characteristic: from the given line a, with each of points M, we always define an only point M' symmetrical to point M through a. We define the conception as follows:

The symmetry with axis a, called S_a, is a transformation that maps each of points M onto point M' as follow : If $M \in a$ then $M' \equiv M$; if $M \in a$ then M' is symmetric to the point M with respect to a. Line a is called the axis of symmetry or symmetric axis.

3. Leaning theorems according to constructivism with the help of dynamic geometry software

Using dynamic geometry software we can move figures, verify and predict the results of problems. Therefore, it gives us a lot of advantages in leaning theorems. Similar to learning concepts, the process of leaning theorems on transformations according to constructivism is as follows:

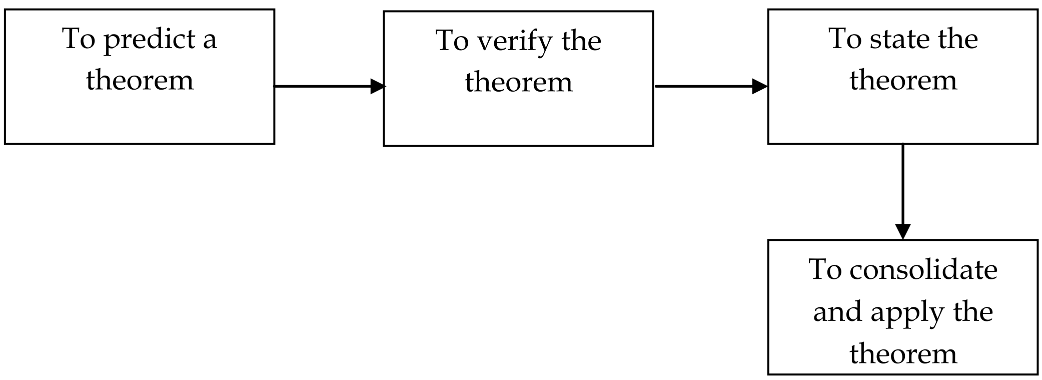


Figure 7. Learning theorems on transformations according to constructivism

3.1. To predict a theorem

At this step, teachers use dynamic geometry software to move reflexes, define corresponding images or set up the equation of the images and the qualitative and quantitative relation between images and reflexes.

Example 6

Given line a and two distinct points M, N. Let M', N' be images of M, N in the symmetry S_a. When M, N move on the plane, compare MN with $M'N'$.

The manipulations of theorem prediction on dynamic geometry software are as follows:

- Draw line a.

- Draw two points M, N.

- Draw two images M', N' of M, N in the symmetry S_a.

- When we move M, N, we see that M', N' move and $MN = M'N'$.

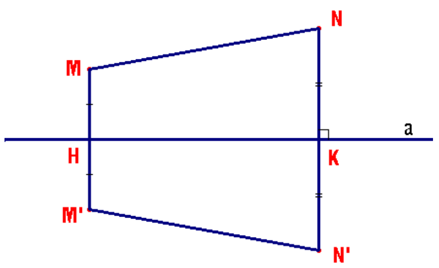


Figure 8

3.2. To verify the theorem

Different from other learning methods, the verifiable step of the theorem aims at proving a given statement or a statement that learners believe true. This step of learning based on constructivism is to verify the predictions of learners that learners do not assert. In this step, teachers use the following methods:

- *Method 1*: have students draw figures, discuss them and choose the symbols on figures; motivate students to write the hypothesis and conclusion of the theorem.

- *Method 2*: motivate students to exchange their points of view for the predictions of proof methods by:

+ discussing in order to find out the similarity between the required proof and the given proof, from which we can find out the sub-lines, or predict the solutions.

+ Having students practise the converse reasoning since this is an effective way of proving the theorem and it helps learners to develop thoughts.

+ Having students draw figures, discuss them and choose symbols on figures, as well as motivate students to write the hypothesis and conclusion of the theorem. Teachers should be interested in the predictions of learners visually.

- *Method 3*: give students opportunities to see the problem by using different methods.

Example 7

Given line a and two distinct points M, N. Let M', N' be the images of M, N in S_a. Prove that $MN = M'N'$.

Solution 1

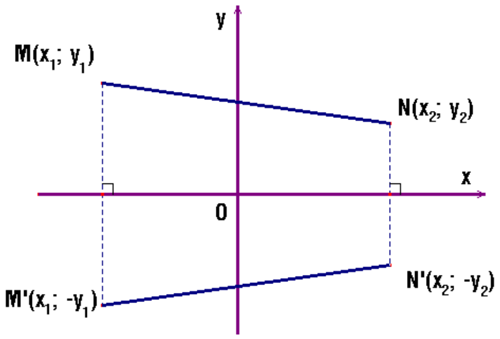


Figure 9

Take the system of coordinates such that the x-axis is a. The transformation S_{Ox} maps $M(x_1\,;y_1)$ onto $M'(x_1\,;-y_1)$; $N(x_2\,;y_2)$ onto $N'(x_2\,;-y_2)$, and we have:

$$MN = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M'N' = \sqrt{(x_2 - x_1)^2 + (-y_2 + y_1)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$

Thus $M'N' = MN$.

Solution 2

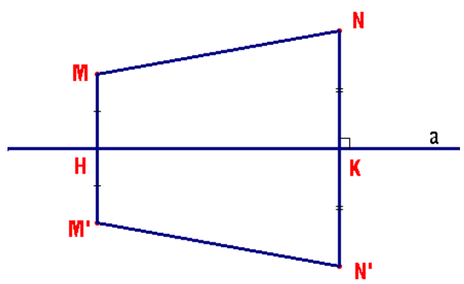


Figure 10

$$MN^2 = \overrightarrow{MN}^2 = \left(\overrightarrow{MH} + \overrightarrow{HK} + \overrightarrow{KN}\right)^2$$

Since $\overrightarrow{MH}$ is direction with $\overrightarrow{KN}$, so $\overrightarrow{MH}=m\overrightarrow{KN}$ or

$$MN^2=\overrightarrow{MN}^2=\left(\overrightarrow{HK}+m\overrightarrow{KN}+\overrightarrow{KN}\right)^2$$

$$=\left(\overrightarrow{HK}+(m+1)\overrightarrow{KN}\right)^2$$

$$=\overrightarrow{HK}^2+2(m+1)\underbrace{\overrightarrow{HK}.\overrightarrow{KN}}_{0}+(m+1)^2\overrightarrow{KN}^2=\overrightarrow{HK}^2+(m+1)^2\overrightarrow{KN}^2 \quad (1)$$

$$M'N'^2=\overrightarrow{M'N'}^2=\left(\overrightarrow{M'H'}+\overrightarrow{HK}+\overrightarrow{KN'}\right)^2 \quad (\overrightarrow{M'H}=-\overrightarrow{MH},\ \overrightarrow{KN'}=-\overrightarrow{KN})$$

$$=\left(\overrightarrow{HK}-(\overrightarrow{MH}+\overrightarrow{HK})\right)^2=\left(\overrightarrow{HK}-(m+1)\overrightarrow{KN}\right)^2$$

$$=\overrightarrow{KH}^2-2(m+1)\underbrace{\overrightarrow{HK}.\overrightarrow{KN}}_{0}+(m+1)^2\overrightarrow{KN}^2$$

$$=\overrightarrow{KH}^2+(m+1)^2\overrightarrow{KN}^2 \quad (2)$$

Since (1) and (2): $M'N'^2=MN^2 \Rightarrow M'N'=MN$.

3.3. To state the theorem

The symmetry preserves the distance between two arbitrary points.

3.4. To practise and apply the theorem

After proving the theorem, we apply it to proving the following corollary.

The symmetry maps three points A, B, C colinear with B, between A and C onto three points A', B', C' colinear with B', between A' and C'.

Solution 1

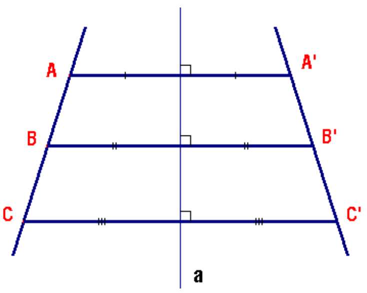


Figure 11

Three given collinear points A, B, C; B is between A and C.

Let $\begin{cases} A' = S_a(A) \\ B' = S_a(B) \\ C' = S_a(C) \end{cases}$

According to the above theorem:

$$\begin{cases} A'B' = AB \\ B'C' = BC \quad (1) \\ A'C' = AC \end{cases}$$

Since A, B, C are collinear, B is between A and C, so :

$AB + BC = AC$ (2). From (1) and (2), we follow that: $A'B' + B'C' = A'C'$ (3)

The equality (3) proves that A', B', C' are collinear and B' is between A' and C'.

Solution 2

Take the system of coordinates Oxy so that Ox is coincident with a. Suppose that $A(x_A ; y_A), B(x_B ; y_B), C(x_C ; y_C)$.

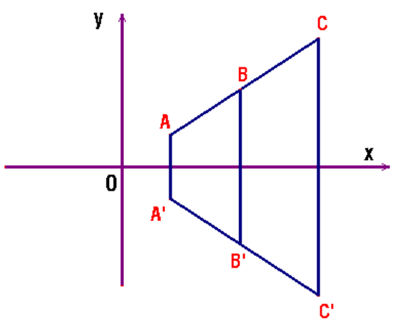


Figure 12

We have:

$$S_a : \begin{matrix} A(x_A; y_A) \mapsto A'(x'_A; y'_A) \\ B(x_B; y_B) \mapsto B'(x'_B; y'_B) \\ C(x_C; y_C) \mapsto C'(x'_C; y'_C) \end{matrix}$$

So $\begin{cases} x'_A = x_A \\ y'_A = -y_A \end{cases} \Rightarrow A'(x_A; -y_A)$; $\begin{cases} x'_B = x_B \\ y'_B = -y_B \end{cases} \Rightarrow B'(x_B; -y_B)$

$\begin{cases} x'_C = x_C \\ y'_C = -y_C \end{cases} \Rightarrow C'(x_C; -y_C)$.

Since A, B, C are collinear, with B between A and C, $\overrightarrow{AB} = k\overrightarrow{AC}$ with $0 < k < 1$ (1).

$$\Leftrightarrow \begin{cases} x_B - x_A = k\left(x_C - x_A\right) \\ y_B - y_A = k\left(y_C - y_A\right) \end{cases} \quad (2)$$

We have: $\overrightarrow{A'B'} = \left(x_B - x_A; -y_B + y_A\right) = \left(x_B - x_A; -\left(y_B - y_A\right)\right)$

$$\overrightarrow{A'C'} = \left(x_C - x_A; -y_C + y_A\right) = \left(x_C - x_A; -\left(y_C - y_A\right)\right) \qquad (3)$$

From (2): $\overrightarrow{A'B'} = \left(k\left(x_C - x_A\right); -k\left(y_C - y_A\right)\right) = k\left(x_C - x_A; -\left(y_C - y_A\right)\right)$ (4)

From (3) and (4): $\overrightarrow{A'B'} = k\overrightarrow{A'C'}$ with $0 < k < 1 \Rightarrow A', B', C'$ are collinear, with B' between A' and C'.

4. Learning to solve a problem using constructivism with the help of dynamic geometry software

The knowledge and skills of students at upper secondary school are formed and developed mainly by solving problems. However, when students solve a problem, they get primary difficulty finding the way to deal with it? How to predict the result of the problem? In traditional learning, teachers usually instruct students to think by imagining in mind, with high abstract, so students learn the knowledge with difficulty. Thus, finding an effective way of learning mathematics is always necessary. Using dynamic geometry software opens a new approach in learning to solve problems. Dynamic geometry software helps to build the process of solving problem. The process of solving transformation problem according to the constructivism with the help of dynamic geometry software (see [13]):

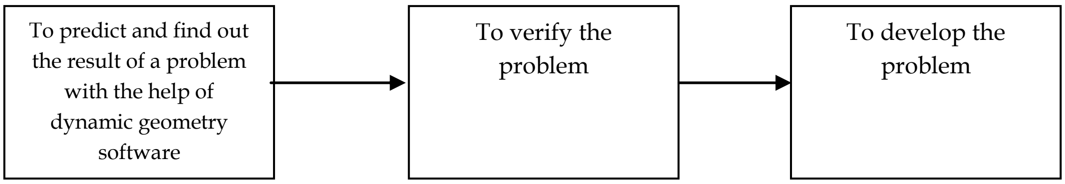


Figure 13. The process of solving problem according to constructivism

4.1. Finding out and predicting the result of a problem with the help of dynamic geometry software

Example 8

Let A_1 be a fixed point, C_1D_1 slides on line d and its length is constant. Find out the position of C_1D_1 so that the perimeter of triangle $A_1C_1D_1$ is minimal.

We use dynamic geometry software to find out the position of C_1D_1 so that the perimeter of triangle $A_1C_1D_1$ is maximal, as follows:

Step 1: Construction

. Draw point A_1.

. Draw line d.

. Draw point C_1.

. Draw C_1D_1 on d so that the length of C_1D_1 is constant.

. Draw triangle $A_1C_1D_1$ by the tool Polygon.

. Measure the perimeter of triangle $A_1C_1D_1$.

. Draw the system of coordinates Oxy so that the x-axis Ox (Ox containing d) and the y-axis Oy are perpendicular.

. Draw N on Oy so that ON equals to the perimeter of triangle $A_1C_1D_1$.

. Draw a straight line passing through point N and perpendicular to Oy, cutting the straight line passing through point C_1 and perpendicular to Ox at E.

Step 2: Making trace

. Making trace to point E, moving point C_1, we obtain the trace of point E that needs to be found.

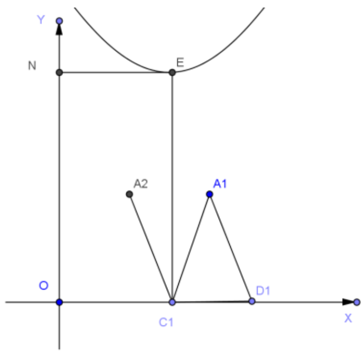


Figure 14

Moving point C_1 to the position so that the ordinate of point E is minimal, we see that triangle A_1C_1D is an isosceles triangle at A_1.

4.2. Verifying a problem

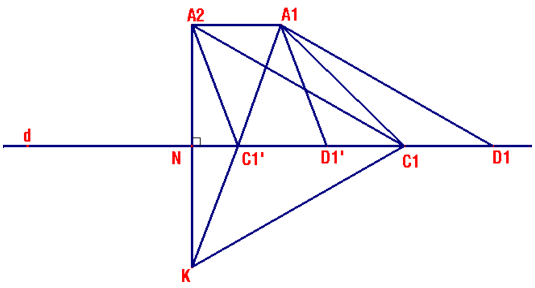


Figure 15

In order to prove the perimeter of triangle $A_1C_1D_1$ is minimal, we need to prove that $A_1C_1 + A_1D_1$ is minimal.

Using the translation $\overrightarrow{D_1C_1}$, we have $A_1 \mapsto A_2$.

Let K be a point symmetric to the point A_2 with respect to the line d. Connect KA_1, it meets line d at $C_1^{'}$.

Use the translation $\overrightarrow{A_2A_1}$, we have $C_1^{'} \mapsto D_1^{'}$.

We have $C_1A_1 + A_1D_1 \geq C_1A_1 + C_1A_2 = C_1A_1 + C_1K \geq A_1K = A_1C_1^{'} + C_1^{'}A_2 = C_1^{'}A_1 + D_1^{'}A_1$.

The equality occurs if C_1 is coincident with $C_1^{'}$.

4.3. Develop the problem

We develop the above problem into the following one:

Example 9

Let A, B be two fixed points and they lie on the same side of the line d. CD slides on d so that its length is constant. Find the position of CD so that the perimeter of quadrilateral $ABCD$ is minimal.

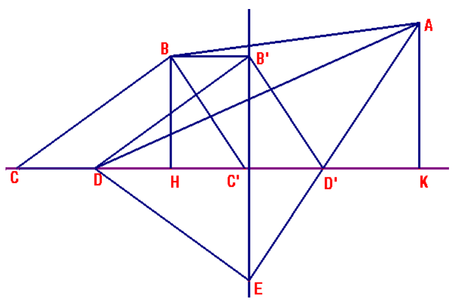


Figure 16

Use the translation $T_{\overrightarrow{CD}} : B \mapsto B'$.

The perimeter of quadrilateral $ABCD$ is minimal if $BC + AD$ is minimal. It means that $B'D + AD$ is minimal.

Similar to problem 1, this sum is minimal if point D is coincident with D' being the point of intersection of EA and d (E is the point symmetric to point B' with respect to d).

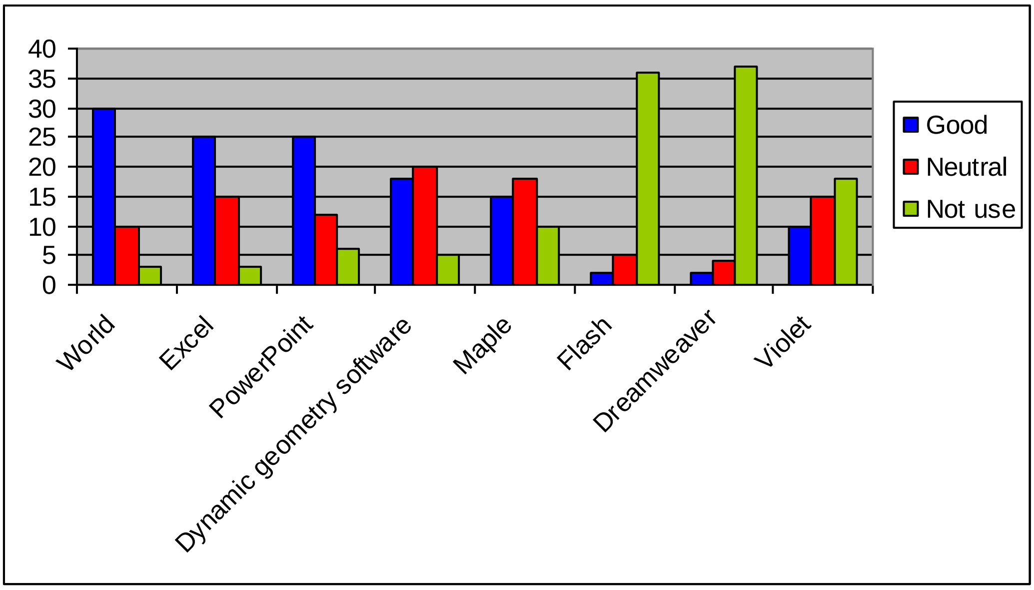
Use the translation $T_{\overrightarrow{DC}} : D' \mapsto C'$, we have $BC + AD = B'D + AD = DE + AD \geq AE \Leftrightarrow BC + AD \geq B'D' + AD' = BC' + AD'$.

The equality occurs if CD is coincident with $C'D'$.

5. Results and discussion

We delivered survey forms to 43 teachers of upper secondary schools in Ho Chi Minh city, Viet Nam in order to check the applying informatics in learning mathematics. The result is as follows:

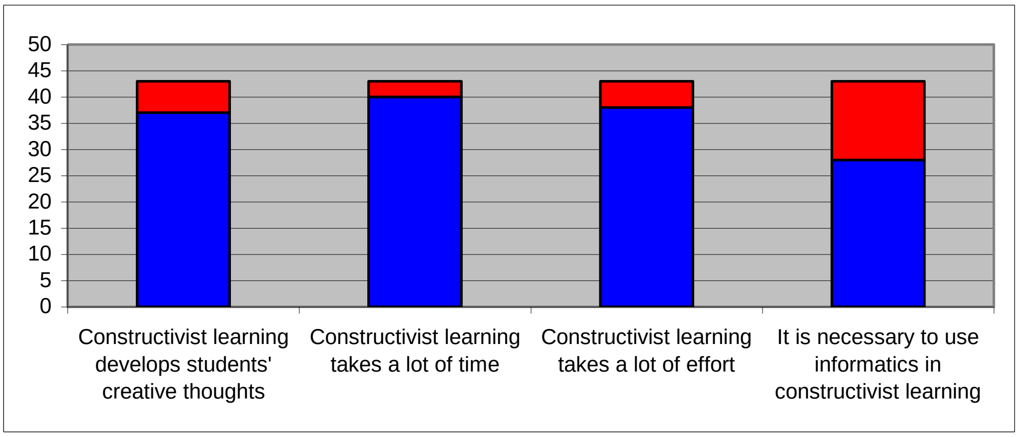
Chart 1.1. The software that teachers may apply in learning mathematics



From the above chart, we see that the number of teachers knowing dynamic geometry software is high. The number of teachers who are proficient in understanding and using dynamic geometry software is also rather high.

We also delivered survey forms to teachers on the necessity of using constructivism in learning mathematics. The result is as follows:

Chart 1.2. The ideas of teachers on constructivist leanring



From the chart, we see the necessity of applying constructivism in learning mathematics to develop students' thought. However, the weak point of this method is that it takes a lot of time and effort of teachers.

We also delivered survey forms to 243 students at upper secondary schools in Ho Chi Minh city, Vietnam in order to check the applying dynamic geometry software in learning mathematics. The result is as follows:

Chart 1.3. The ideas of students on applying dynamic geometry software in constructivist learning

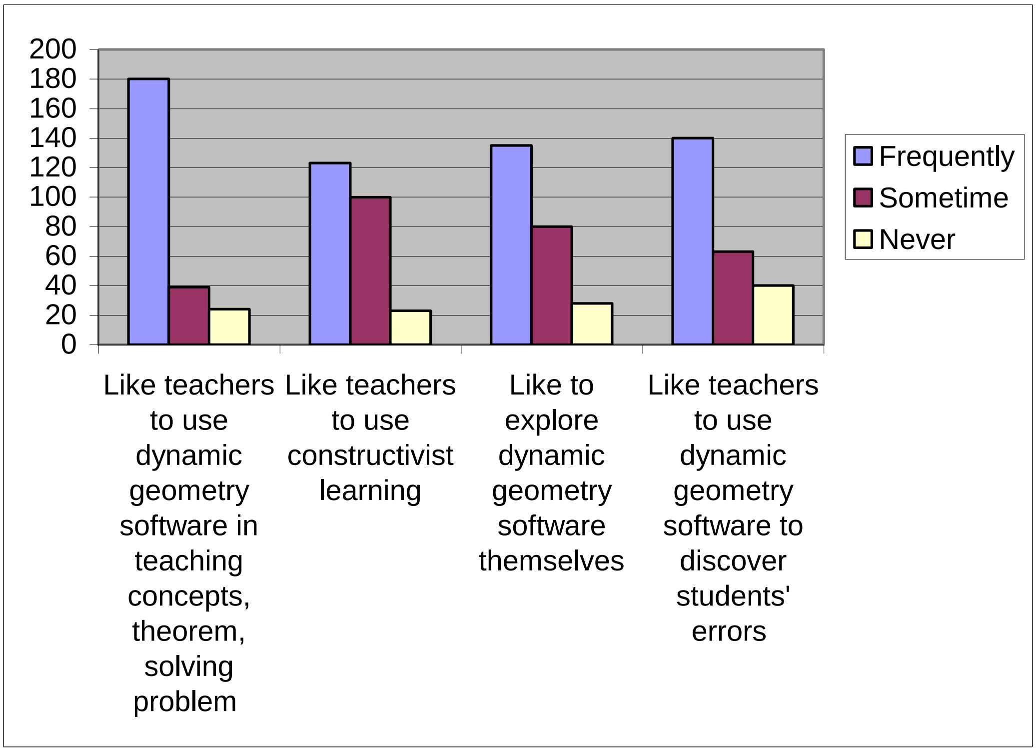


Chart 1.3 shows that students like teachers to use dynamic geometry software in teaching as well as learning using constructivism. Students are interested in dynamic geometry software as well as wanting teachers to point out the mistakes in solving problems. The number of students using dynamic geometry software in self study is high. From this, we can assert that using dynamic

geometry software in teaching according to constructivism makes students interested in learning. Constructivist method is better than traditional method.

6. Conclusion

We can say that, the process of constructivism of new knowledge is the process in which we gain new knowlege from acquired knowledge. It is the process connecting two poles "known" with "will-know" of the awareness. Thus, when teachers use constructivism, they should know how to divide knowledge and skills into small units so that students can solve a problem by themselves. Teachers should know how to organize, instruct students to survey, find out, and solve each part of knowledge and new skills. The process of constructivism of new knowledge motivates independent and creative thoughts of students. The applying of dynamic geometry software in learning plane geometry is also a new way of helping students to understand lessons and to develop skills in informatics. Students can use dynamic geometry software to find out the solution, the incorrectness of a problem. In addtion, the software also helps student seff-learn. Students like to have a passion for learning with the help of informatics.

References

Dao, H. T. (2006). Applying informatics in learning chemistry according to constructivism, *Vietnamese Educational Journal*, 35-37.

Cakir, M. (2008). Constructivist approaches to learning in science and their implications for science pedagogy: A literature review, *International journal of environmental & science education*, 193-206.

Cao, T. H. (2007). Learning concepts for students at upper secondary school according to constructivism, *Vietnamese Educational Journal*, 29-30.

Cao, T. H, (2008). Learning mathematical theorem at upper secondary schook according to constrcutivism, *Vietnamese Educational Journal*, 33-36.

Gordon, M. (2009). Toward a pragmatic discourse of constructivism: Reflections on lessons from practice, *Educational studies*, 39-58.

Karagiorgi, Y., & Symeou, L. (2005). Translating constructivism into instructional design : Potential and limiations, *Educational technology & Society*, 17-27.

Liu, C. H., & Matthews. R (2005). Vygotsky's philosophy: Constructivism and its criticism examined, *International Educational journal*, 386-399.

Doan, Q., Van, H, C., Pham, K, B., & Ta, M. (2007). *Geometry 11 (Advanced)*, Education Publishing House (Book written in Vietnamese), 3-37.

Do, T. T. (2009), Building and organizing some learning situations according to constructivism, *Vietnamese Educational Journal*, 43-44.

Do, T. D., & Vu, V. D. (2005). Applying constructivism in learning mathematics at primary school, *Vietnamese Educational Journal*, 26 – 27.

Nguyen H. C., & Cao, T. H (2003). Teaching at upper secondary school according to constructivism, *Vietnames Educational Journal,* 28-29.

Nguyen, Q. L. (2007). Applying constructivsim in the inovation of physical leanring method, *Vietnamese Educational Journal,* 32-34.

Pham, S. N (2008). Improving the effectiveness of learning analysis for students at upper secondary school according to constructivism, *Vietnamese Educational Journal,* 42-43.

Phi, T. T. V (2009). The approach of constructivism in learning solving problem at secondary school, *Vietnamese Educational Journal,* 42-43.

Pho, D, H (2013). *P*roblem-base learning at primary school according to constructivism, *Vietnamese Educational Journal,* 15-17.

Tran, V. H., Nguyen M. H., Khu Q. A., Nguyen, H. T., & Phan, V. V. (2007), *Geometry 11,* Education Publishing House (Book written in Vietnamese), 4-40.