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Influence of Mathematical Representation and Mathematics Self-Efficacy on the Learning Effectiveness of Fifth Graders in Pattern Reasoning

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Abstract. The aim of this study was to examine the influence of mathematics self-efficacy and diverse mathematical representations in learning materials on the performance and learning attitude of elementary school learners with regard to pattern reasoning. The research samples comprised one hundred and fifty fifth-grade students from an elementary school in Central Taiwan. We adopted a two-factor quasi-experimental design with mathematical representation and mathematics self-efficacy as the independent variables. Digital learning materials were graphical or numerical and the learners designated as having high or low mathematics self-efficacy. The dependent variables included performance of pattern reasoning and attitudes towards learning mathematics. The former was divided into number sequence reasoning and graphic sequence reasoning, whereas the latter included learning enjoyment, motivation, and anxiety. The research findings indicate that (1) using graphical learning materials enhances performance in pattern reasoning; (2) using digital learning materials in teaching can improve attitudes towards learning mathematics; (3) learners with high mathematics self-efficacy display more positive views towards learning mathematics.

Keywords: pattern reasoning; representation; mathematics teaching; digital learning materials; mathematics self-efficacy

1. Introduction

In mathematics, pattern reasoning is generally a difficult topic for elementary school learners. Learners often fail to perceive pattern relationships and internalize them into personal knowledge and understanding, which then leads to inflexibility in their approach to mathematical problems (Lee, Chen & Chang, 2014). As the thinking patterns of elementary school learners are still in the concrete operational stage, they require manipulable objects, the enactive and iconic representation of which helps learners make connections with previously-acquired knowledge. Providing learners with concrete representations on interactive digital platforms can thus assist them in translating concrete into abstract thinking.

The learning of pattern reasoning generally begins with inductive reasoning related to quantitative relationships before progressing on to deductive reasoning. These higher levels of logical thinking often involve abstract concepts, which learners must represent with concrete objects or appropriate symbols. Lewis and Mayer (1987) indicated that most difficulties in problem-solving occur in the representation stage. As a result, the process of translating problems into internal representations is the key to whether learners can successfully solve a problem. If learners can understand different forms of conversion processes for mathematical representation, they will be able to grasp the mathematical concepts involved.

The self-efficacy of learners is also a factor of learning effectiveness, and mathematics is no exception. Learners with greater mathematics self-efficacy have more confidence and better learning effectiveness in mathematics as well as less mathematics anxiety (Lee & Chen, 2015; Hackett & Betz, 1989; Schunk, 2007). The means of enhancing the mathematics self-efficacy of learners is thus an issue worth investigating. We used digital learning materials designed for diverse mathematical representation with the objectives of improving the performance of fifth graders in pattern reasoning and their attitudes toward learning mathematics. During this process, we examined the influence of various mathematical representations and degrees of mathematics self-efficacy on the performance of learners in pattern reasoning and their attitude towards learning mathematics and determined whether interaction effects exist between mathematics self-efficacy and mathematical representations.

2. Literature Review

We investigated the influence of different mathematical representations and degrees of mathematics self-efficacy on the performance of learners in pattern reasoning and their attitude towards learning mathematics from the perspective of mathematics teaching and the incorporation of information technology into teaching. We thus collated relevant literature associated with pattern reasoning, mathematical representations, and mathematics self-efficacy.

2.1 Pattern reasoning

The essence of mathematics is seeking patterns and relationships among them (Lee & Chen, 2009). With the experience accumulated from pattern reasoning, one can learn the means of perceiving and generalizing quantitative patterns in objects and matters to set up and solve mathematical problems. Blanton and Kaput (2002) stated that behind any special phenomenon lies the basis and pattern of its occurrence.

Pattern reasoning activities not only emphasize inductive reasoning beginning from quantitative patterns but also extend to deductive reasoning activities (Fernandez & Anhalt, 2001). This means that learners identify and confirm patterns before further generalizing the patterns for problem solving. Owen (1995) divided mathematics patterns into three types: repeating patterns, structural patterns, and growing patterns, all of which are present in the elementary school mathematics curriculum in Taiwan.

2.1.1 Repeating patterns

As the name suggests, repeating patterns evidence cycles or repetition (Owen, 1995) of specific characteristics such as colors, shapes, directions, sizes, sounds, or numbers, for example, "yellow, green, red, yellow, green, red," and "□, ○, △, □, ○, △".

2.1.2 Structural patterns

Structural patterns imply the presence of certain characteristics within a group, for example, compositions of 5 (4 + 1, 3 +2, 2 +3, and 1 + 4). In elementary school mathematics, the commutative laws, the associative laws and the distributive laws of multiplication and addition are all topics involving structural patterns. For example, 3×5=15 and 5×3=15, or 8×4=(5+3)×4=(5×4)+(3×4).

2.1.3 Growing patterns

Growing patterns involve changing the form of a number through predictive methods. Owen (1995) categorized the contents of growing patterns as sequences, which are lists of non-repetitive numbers that expand according to a single rule. In formal curriculum activities, number sequences are the most typical type of sequences, encompassing arithmetic sequences, geometric sequences, and Pascal's triangle. For instance, the sequence 5, 10, 15, 20… increases by 5 with every term.

2.2 Mathematical representation

Mathematical representations are defined as the different forms of representations that learners use to interpret a problem (Ainworth, 2006). The National Council of Teachers of Mathematics (NCTM) (2000) identified mathematical representations as depictions of mathematical concepts formed by learners, indicating their understanding and application of said concepts. Mathematical representations therefore play an important role in the formation of mathematical concepts. Through different representations, learners learn mathematics and gain knowledge. Bruner (1966) claimed that the process of conceptual development is the formation of a system of representations; he divided learning into three development processes involving enactive, iconic, and symbolic representations. Heddens (1984) divided learning stages into concrete, semi-concrete, semi-abstract, and abstract representations and stated that learners must first be able to internalize new knowledge in the concrete stage before systematically assigning abstract representations to the new knowledge. By creating sound connections between the real world and the abstract world, they build solid foundations for mathematical thinking.

Kaput (1987) sought to explain the link between mathematical representation and mathematics learning, proposing four categories of the former: cognitive and perceptual representation, explanatory representation, representation within mathematics, and external symbolic representation. Janvier (1987) showed that external symbolic representations influence as well as reflect the internal representations of the mathematical knowledge possessed by learners. Based on the perspective of communication, Lesh, Post, and Behr (1987) classified five different types of representations: real script, manipulative models, static pictures, spoken language, and written symbols. They stressed the importance of conversions between representations, which means that learners of mathematics must be able to understand diverse forms of representation, move easily between forms of representations, and select the most appropriate and convenient method of representation to explain and solve problems.

Willis and Fuson (1988) found the use of pictorial representations in teaching second graders to solve word problems in addition and subtraction to be effective. Tchoshanov (1997) carried out a pilot experiment on trigonometric problem solving and proof for high school students in Russia. The analytic group was taught by a traditional algebraic approach. The visual group was taught by a visual approach using enactive (i.e., geoboard as manipulative aid) and iconic (pictorial) representations. The representational group was taught by a combination of analytic and visual means. The results showed that the representational group had a better learning performance than the visual and analytic groups. Therefore, we understood that any intensive use of only one specfic mode of representation does not enhance students' conceptual understanding and representational thinking.

2.3 Mathematics self-efficacy

Self-efficacy is a determinant influencing the learningeffects in mathematics and can be used to accurately predict learning achievements in mathematics. Hackett and Betz (1989) established significant and positive correlations among learning effectiveness, self-efficacy, and learning attitudes in mathematics. Anjum (2006) further indicated a positive correlation between self-efficacy and mathematics achievements on every grade level of elementary school, the degree of which increased with the grade. Skaalvik and Skaalvik's (2006) found that middle school and high school mathematics students showed self-efficacy predicted subsequent learning performance more accurately than prior achievement. They found that self-efficacy mediated academic achievement. Mathematics achievement is influenced significantly by student's attitudes and self-efficacy. Lee and Cheng (2012) also found that students with high mathematics self-efficacy have better learning outcomes and attitudes toward mathematics than those with low mathematics self-efficacy when learning equivalent fractions. Therefore, enhancing the mathematics self-efficacy of learners can benefit their effectiveness in learning mathematics.

3. Methodology

3.1 Research Participants

In this study, we targeted fifth-grade elementary learners. The research samples comprised four fifth-grade classes from an elementary school in Central Taiwan. Before conducting the experiment, the participants were randomly assigned to two groups: one using graphical learning materials and the other using numerical learning materials. A mathematics self-efficacy scale was used to assign the top 45 % and the bottom 45 % of the learners as those with high and low mathematics self-efficacy, respectively. We derived a total of 121 valid samples.

3.2 Research Instruments

3.2.1 Mathematics self-efficacy scale

The purpose of applying a mathematics self-efficacy scale was to assess whether the learners had the confidence to effectively execute mathematical learning activities. We adopted the mathematics self-efficacy scale revised by Lee and Cheng (2012) from the General Self-efficacy subscale developed by Sherer and Maddux (1982). The scale comprises three constructs: initiation, persistence, and self-confidence. Initiation involves the degree of confidence that a learner has to initiate learning when encountering a new mathematical learning activity or a more difficult mathematical task; persistence indicates the degree of confidence that a learner has to persist in learning when experiencing setbacks; and self-confidence refers to the degree of confidence that a learner has in completing tasks. Each construct contained 6 question items, accounting for a total of 18 question items in the scale. We adopted a five-point Likert scale, ranging from strongly disagree (1) to strongly agree (5). Higher scores represented greater mathematics self-efficacy, meaning that learners had greater confidence in their ability to effectively execute mathematical learning activities. An internal consistency test on the reliability of the scale presented an overall Cronbach' s α of 0. 95 – an ideal internal consistency coefficient.

3.2.2 Pattern reasoning materials

AMA (Activate Mind Attention) is a widely known software program in Taiwan that utilizes PowerPoint as a platform for the design and presentation of media for mathematical instruction (Lee &Chen, in press). It is available free of charge from http://ama.nctu.edu.tw/index.php, and its core functions include the structural cloning method (SCM) and trigger-based animation (TA). SCM uses the concepts of structure and cloning to interpret shapes. Its original purpose was to resolve positioning issues in the design of teaching materials, but its ability to imitate paintings of natural landscapes, and create complex symmetrical compositions and spot series ensure a wide range of potential applications. In TA, certain objects serve as buttons that control series of animations. TA can assist users in displaying digital content to attract the attention of the audience, guide cognition, and reduce cognitive load.

For the contents of the learning materials used in this study, we referred to the curriculum regarding number sequences and graphic sequences in mathematics textbooks published by Kang Hsuan Educational Publishing Group. We used AMA to design the digital materials, which were then reviewed and revised by elementary school teachers and experts who are professional at this topic. The primary learning objective in this topic is to perceive simple quantity patterns and solve problems through concrete observation and exploration, and make connections with three other learning areas in mathematics: numbers and quantities, elementary algebra, and connection. The materials presented four teaching foci in a progressive manner: sequences of odd numbers and even numbers, triangular numbers, square numbers, and Fibonacci numbers. We created materials and worksheets to act as step-by-step guides to exploration of pattern reasoning. Learning objectives were set for each focus based on the curriculum, and the learning achievements based on these objectives were explained in detail. The designs of the digital materials in the numerical learning materials group and the graphical learning materials group were different only in the manner of mathematical representation; the remainder of the contents was the same.

Numerical learning materials

These materials used numerical representations. Aided by worksheets, the teacher presented the foci of the learning materials one by one. Figure 1 shows an example with the square number sequence 1, 4, 9, 16, …. The learners are asked to identify the seventh item and find the pattern. With the interactive buttons in the digital materials, the teacher guided the learners' exploration of the relationship among the numbers, identifying the next item first before finding the seventh with the perceived pattern and recording the ideas on the worksheet.

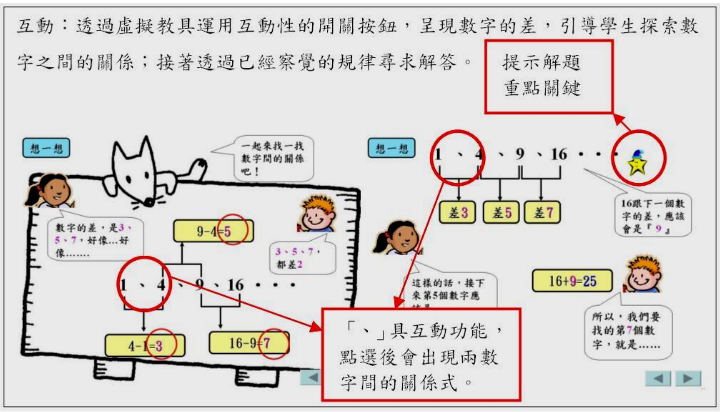

Figure 1: Interactive materials showing the pattern of a square number sequence

Graphical learning materials

These materials used graphical representations. Aided by worksheets, the teacher presented the foci of the learning materials one by one. Figure 2 displays the graphical example of square numbers, also asking the learners to identify the seventh item and find the pattern. With the interactive buttons in the digital materials, the teacher presented the graphical changes and guided the learners' exploration of the relationship among the graphs, identifying the graph of the next item first before finding the seventh with the perceived pattern and recording the ideas on the worksheet.

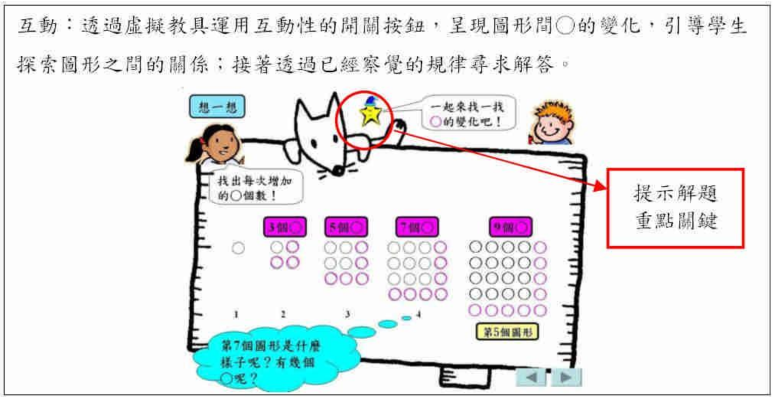

Figure 2: Solving the square number pattern with graphics

3.2.3 Pattern reasoning achievement test

The aim of the pattern reasoning achievement test was to assess the performance of the learners in pattern reasoning after the teaching experiment using digital materials with different mathematical representations. Based on the teaching contents and the studies conducted by Rivera and Becker (2008), the test was divided into two portions: number sequences and graphic sequences. Each portion contained five problems with 1 point for each problem, resulting in a total score of 10 points.

Number sequence reasoning was assessed by evaluating the learners' ability to seek patterns among numbers and solve number sequence problems. The problems involved (1) arithmetic sequences and (2) second-order arithmetic sequences, both of which were presented with number sequences. Graphic sequence reasoning was assessed by evaluating learners' ability to seek patterns among graphs and solve graphic sequence problems. The problems involved (1) arithmetic sequences and (2) second-order arithmetic sequences, both of which were presented with graphic sequences.

An internal consistency test on the reliability of the pattern reasoning achievement test yielded a Cronbach's α of 0.71 in the number sequence reasoning portion, a Cronbach's α of 0.73 in the graphic sequence reasoning portion, and a Cronbach's α of 0.83 for the entire test, indicating acceptable internal consistency. The difficulty indexes of the problems ranged between 0.30 and 0.86, whereas the discrimination indexes of the problems ranged between 0.35 and 0.95. On the whole, the difficulty index and discrimination index in the pattern reasoning achievement test were appropriate.

3.2.4 Attitudes towards learning mathematics questionnaire

A questionnaire was used to understand the feelings of the learners as they learned the concepts of number and graphic sequences using different mathematical representations. We adopted the questionnaire created by Lee and Cheng (2012), which is divided into three aspects: enjoyment of and motivation and anxiety toward learning. Each aspect contains 5 question items, accounting for a total of 15 question items. We utilized a five-point Likert scale, ranging from strongly disagree (1) to strongly agree (5). Questions related to enjoyment and motivation were positive, whereas those regarding learning anxiety were negative. In the positive items, the subjects choosing 1 were given 1 point, those choosing 2 were given 2 points, and so on. In the negative items, the scores were the opposite. Higher total scores indicated more positive attitudes towards learning mathematics. The Cronbach's α of the entire questionnaire was 0. 74, representing acceptable internal consistency.

4. Results and Discussion

4.1 Analysis of learning performance in pattern reasoning

We analyzed the performances of learners in both number sequence reasoning and graphic sequence reasoning. The means and standard deviations of the two sets of scores are showed in Table 1. The performance of the students in the graphical learning materials group was better than that of the students in the numerical learning materials group. Also, students with high mathematics self-efficacy displayed better performance in pattern reasoning than those with low mathematics self-efficacy.

Table 1: Summary of learning performance results

Pattern reasoning construct	Group	Mean	Standard deviation	Number of subjects
Number sequence reasoning	Numerical learning materials	2.86	1.212	56
	Graphical learning materials	3.29	1.027	65
	High mathematics self-efficacy	3.12	1.195	60
	Low mathematics self-efficacy	3.07	1.078	61
Graphic sequence reasoning	Numerical learning materials	2.48	1.375	56
	Graphical learning materials	3.02	1.192	65
	High mathematics self-efficacy	2.93	1.313	60
	Low mathematics self-efficacy	2.61	1.282	61

4.1.1 Analysis of performance in reasoning with number sequences

Table 2 displays a summary of the ANOVA regarding number sequence reasoning. The interaction effect between mathematical representation and mathematics self-efficacy did not reach the significance ($F(1, 117)= 0.159$, $p= 0.908$). The main effect of mathematical representation was significant ($F(1, 117) = 4. 439$, $p = 0.037$), whereas the main effect of mathematics self-efficacy was not ($F(1,117) = 0.018$, $p = 0.894$). The mean score in number sequence reasoning shows that the students were more receptive to the graphical learning materials (mean= 3.29) than they were to the numerical learning materials (mean= 2.86). In addition, the students with high mathematics self-efficacy exhibited no differences in number sequence reasoning from those with low mathematics self-efficacy.

Table 2 Summary of ANOVA for number sequence reasoning

Source of variance	SS (Type-III sum of squares)	Df (Degree of freedom)	MS (Sum of squares)	*F* (F test)	Sig. (Significance)

Mathematical representation	5.626	1	5.626	4.439*	.037
Mathematics self-efficacy	.023	1	.023	.018	.894
Mathematical representation×Mathematics self-efficacy	.017	1	.017	.013	.908
Error	148.266	117	1.267		

4.1.2 Analysis of performance in reasoning with graphic sequences

Table 3 displays a summary of the ANOVA for graphic sequence reasoning. The interaction effect between mathematical representation and mathematics self-efficacy did not reach the significance ($F(1, 117)= 0.226$, $p= 0.635$). The main effect of mathematical representation was significant ($F(1, 117) = 4. 896$, $p = 0.029$), whereas the main effect of mathematics self-efficacy was not ($F(1,117) = 1.517$, $p = 0.221$). These results reveal that the students that had used graphical learning materials (mean= 3.02) performed better in graphic sequence reasoning than those that had used numerical learning materials (mean= 2.48). Furthermore, subjects with high and low mathematics self-efficacy delivered the same level of performance.

The analysis results regarding pattern reasoning performance show that the mathematical representation used in the learning materials had significant influence on the learning performances in number sequence reasoning and graphic sequence reasoning, whereas mathematics self-efficacy did not have significant influence.

The research results demonstrate that the performance displayed by learners learning with graphical materials in pattern reasoning was superior to that displayed by learners learning with numerical materials. One possible explanation was that the graphical materials provided the environment so that the students had more opportunities to have a connection between numerical and graphic representations. This ability to adapt multiple representations to the problem at hand reflects one's grasp of mathematical concepts (Brenner, et al., 1999; Cai, 2001). Therefore, making conversions between different representation systems can assist learners in interpreting problems, enhance their understanding of mathematical concepts, and enable them to make connections with related concepts, all of which make learning mathematics more meaningful.

With regard to mathematics self-efficacy, we discovered no significant differences between learners with high and low mathematics self-efficacy in pattern reasoning. One possible reason was that both groups used dynamic interactive digital learning materials, and both groups were able to observe patterns in numerical and graphic representations to the same extent. Therefore, the digital materials were helpful to learners with either high or low mathematics self-efficacy.

Table 3 Summary of ANOVA for graphic sequence reasoning

Source of variance	SS (Type-III sum of	Df (Degree of	MS (Sum of squares)	*F* (F test)	Sig. (Significance)

	squares)	freedom)			
Mathematical representation	8.033	1	8.033	4.896*	.029
Mathematics self-efficacy	2.489	1	2.489	1.517	.221
Mathematical representation×Mathematics self-efficacy	.371	1	.371	.266	.635
Error	191.944	117	1.641		

4.2 Analysis of attitudes towards learning mathematics

The means and standard deviations of the scores resulting from the mathematics self-efficacy questionnaire are presented in Table 4. A score of 3 indicates a neutral position, and higher scores mean more positive attitudes, which implicate greater enjoyment in and motivation toward learning mathematics as well as less anxiety. The mean scores show that the students displayed positive learning attitudes towards the integration of different mathematical representations in the materials. The graphical learning materials group displayed attitudes that were slightly more positive than the numerical learning materials group. The students also displayed positive learning attitudes regardless of their degree of mathematics self-efficacy, but students possessing high mathematics self-efficacy presented with higher scores than those possessing low mathematics self-efficacy in all three aspects.

Table 4 Summary of results with regard to attitudes towards learning mathematics

Aspect of attitudes towards learning mathematics	Group	Mean	Standard deviation	Number of subjects
Enjoyment in learning	Numerical learning materials	3.400	1.086	56
	Graphical learning materials	3.516	0.985	65
	High mathematics self-efficacy	3.806	1.001	60
	Low mathematics self-efficacy	3.124	0.905	61
Motivation toward learning	Numerical learning materials	3.450	1.204	56
	Graphical learning materials	3.600	0.987	65
	High mathematics self-efficacy	3.966	0.958	60
	Low mathematics self-efficacy	3.102	0.980	61
Anxiety toward learning	Numerical learning materials	3.524	0.787	56
	Graphical learning materials	3.364	1.064	65
	High mathematics self-efficacy	3.714	0.929	60
	Low mathematics self-efficacy	3.168	0.946	61

Table 5 summarizes the ANOVA for attitudes towards learning mathematics. In the enjoyment of and motivation toward learning, the two-dimensional interaction effects between mathematical representation and mathematics self-efficacy reached the level of significance ($F(1,117) = 6.831$, $p= 0.010$; $F(1,117) = 5.400$, $p= 0.022$). This shows that mathematical representation and mathematics self-efficacy exert varying degrees of influence on the enjoyment and motivation of learners in different groups. We then analyzed the simple main effects of mathematical representation and mathematics self-efficacy on the two variables.

Table 5 Summary of ANOVA for attitudes towards learning mathematics

Source of variance	Dependent variable	SS (Type-III sum of squares)	Df (Degree of freedom)	MS (Sum of squares)	*F* (F test)	Sig. (Significance)
Mathematical representation	Learning enjoyment	5.399	1	8.033	.236	.628
	Learning motivation	8.484	1	8.484	.373	.543
	Learning anxiety	27.670	1	27.670	1.342	.249
Mathematics self-efficacy	Learning enjoyment	309.737	1	309.737	13.564*	.000
	Learning motivation	514.294	1	514.294	22.610*	.000
	Learning anxiety	223.604	1	223.604	10.845*	.001
Mathematical representation×Mathematics self-efficacy	Learning enjoyment	155.995	1	155.995	6.831*	.010
	Learning motivation	122.839	1	122.839	5.400*	.022
	Learning anxiety	16.374	1	16.374	.794	.375
Error	Learning enjoyment	2671.813	117	22.836		
	Learning motivation	2661.322	117	22.746		
	Learning anxiety	2412.309	117	20.618		

Figure 3 displays the interaction effects between mathematical representation and mathematics self-efficacy with regard to learning enjoyment. Different mathematical representations caused learners with high mathematics self-efficacy to display significant differences in this variable ($F(1,59) = 4.567$, $p = 0.037$); that is, they had significantly more fun learning with graphical learning materials than with numerical learning materials. In contrast, learners with low mathematics self-efficacy did not display differences in learning enjoyment with regard to mathematical representation ($F(1,60) = 2.383$, $p = 0.128$). Furthermore, in the numerical learning materials group, learners with high mathematics self-efficacy showed no differences from those with low mathematics self-efficacy in this variable ($F(1,54) = 0.407$, $p = 0.526$); however, in the graphical learning materials group, learners with high mathematics self-efficacy had significantly more fun than those with low mathematics self-efficacy ($F(1,63) = 29.060$, $p < 0.05$). The simple main effects analysis of learning enjoyment thus demonstrates that learners with high mathematics self-efficacy have significantly more fun learning mathematics with graphical learning materials than learners using numerical learning materials and learners with low mathematics self-efficacy.

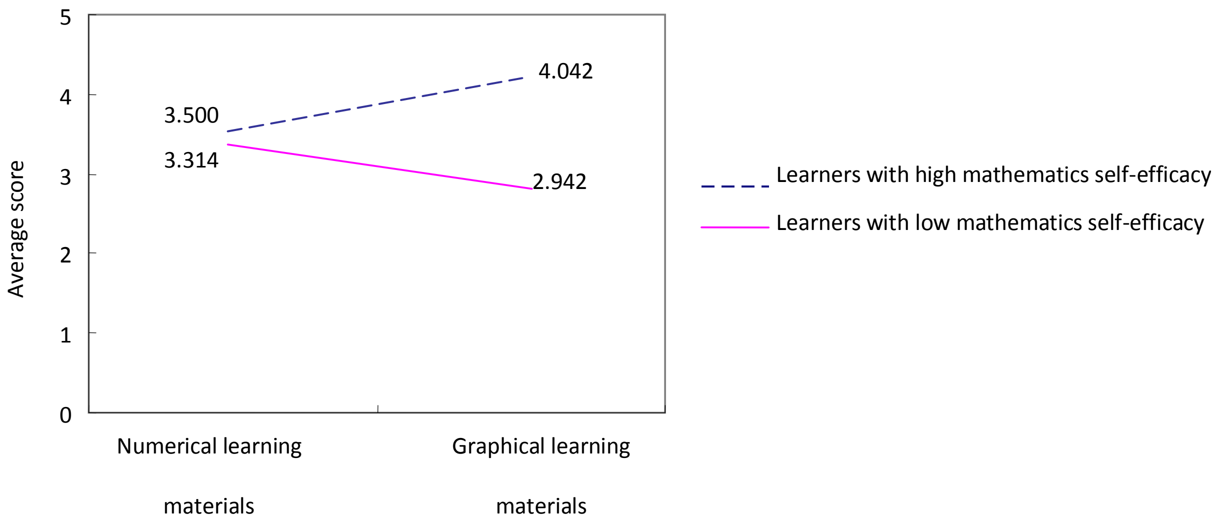

Fig. 3 Interaction effects between mathematical representation and mathematics self-efficacy in learning enjoyment

Figure 4 exhibits the interaction effects between mathematical representation and mathematics self-efficacy with regard to learning motivation. Learners with high mathematics self-efficacy presented significant differences in learning motivation with regard to mathematical representation ($F(1,59) = 4.447$, $p= 0.039$); those learning with graphical learning materials were more motivated than those learning with numerical learning materials. In contrast, learners with low mathematics self-efficacy showed no differences in learning motivation with regard to mathematical representation ($F(1,60) = 1.426$; $p= 0.237$). The degree of mathematics self-efficacy did not have significant influence on students' motivation in the numerical learning materials group ($F(1,54) =1.962$, $p= 0.167$); however, it did have significant influence on learning motivation in the graphical learning materials group ($F(1,63) = 41.275$, $p< 0.05$): learners with high mathematics self-efficacy were significantly more motivated than those with low mathematics self-efficacy. The simple main effects analysis of learning motivation thus reveals that learners with high mathematics self-efficacy are significantly more motivated when learning mathematics with graphical learning materials than learners using numerical learning materials and learners with low mathematics self-efficacy.

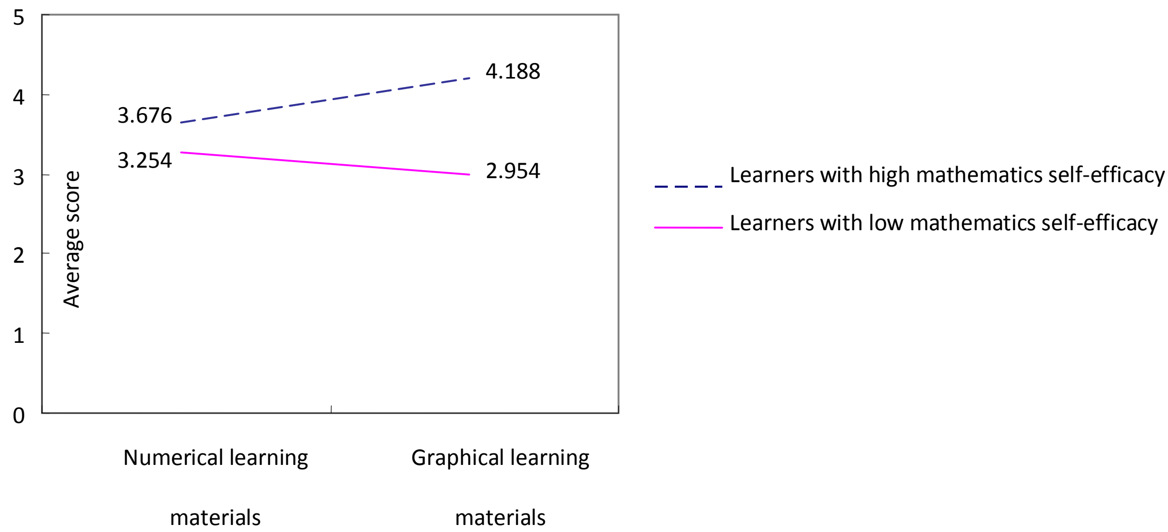

Fig. 4 Interaction effects between mathematical representation and mathematics self-efficacy in learning motivation

In learning anxiety, the main effect of mathematics self-efficacy reached the level of significance ($F(1,117) = 10.845$, $p = 0.001$). The main effect of mathematics self-efficacy and the mean scores of learning anxiety indicate that learners with high mathematics self-efficacy experience less anxiety in learning mathematics than learners with low mathematics self-efficacy. In other words, learners with low mathematics self-efficacy feel more anxious about the learning activities in pattern reasoning.

The analysis results concerning the attitudes towards learning mathematics indicate that when learning with graphical learning materials, learners with high mathematics self-efficacy experience a greater degree of enjoyment and motivation than learners with low mathematics self-efficacy. However, when learning with numerical learning materials, the learners displayed no significant differences in learning enjoyment and motivation related to the degree of mathematics self-efficacy. We speculate that this is because the wealth of information that graphical learning materials provide give learners with high mathematics self-efficacy the confidence to solve the problems without assistance, and they will therefore set more challenging objectives for themselves and work harder in the face of setbacks. As a result, they will have more fun and be more motivated to learn than learners with low mathematics self-efficacy. Numerical representations in learning materials are monotonous and lack the excitement of graphics. For this reason, the students with high mathematics self-efficacy learning with these materials presented no significant differences from those with low mathematics self-efficacy in learning enjoyment or motivation.

Learners with high mathematics self-efficacy feel relatively less anxiety with regard to mathematics than do learners with low mathematics self-efficacy. The students in the numerical and graphical learning groups showed no significant differences in learning anxiety. We conjecture that learners with low mathematics self-efficacy generally believe that their tasks are harder than they really are, that any amount of effort cannot change an established fact, and that their ability to solve problems is insufficient. Such beliefs weaken self-confidence and evoke negative emotional reactions such as anxiety, tension, stress, and depression (Bandura, 1986), all of which cause learners with low mathematics self-efficacy to have greater anxiety in learning mathematics.

The variability and interactivity of the digital learning materials provided in this study make manifest abstract concepts. In addition, as this was the first time the students had used such materials in math class, they were a novelty. As a result, the learners expressed positive feelings regardless of the type of learning material and presented no significant differences in learning anxiety.

5. Conclusions and Suggestions

5.1 Using graphical learning materials can improve the performance of learners in pattern reasoning

In the analysis of performance in number sequence reasoning, the results indicate that learners that had used graphical learning materials obtained better scores than those that had used numerical learning materials. Similar results occurred for graphic sequence reasoning, and both presented significant differences. Therefore, integrating graphical learning materials into teaching can improve the performance of learners in pattern reasoning.

The dynamic and static pictures presented in the graphical learning materials enabled the learners to make connections with previously-acquired knowledge and practice converting from one mathematical representation to another. This kind of flexible use of representation systems is an essential feature of mathematical ability (Dreyfus & Eisinberg, 1996). The answers to the pattern reasoning achievement test also revealed that learners in the graphical learning materials group were more able to describe the patterns that they perceived. In other words, diverse representation during the learning process enhances the understanding of concepts and induces better learning effectiveness in pattern reasoning.

When learners encounter difficulties in mathematics, it is often due to the inability make flexible use of mathematical representations to solve problems. Therefore, teachers should make use of diverse mathematical representations, such as manipulative models, graphs, and abstract symbols, in order to promote independent thinking and holistic understanding rather than the mere use of formulas and algorithms for by-rote problem solving.

5.2 Providing appropriate strategies to enhance mathematics self-efficacy can improve attitudes towards learning mathematics

When learning with graphical learning materials, learners with high mathematics self-efficacy had more fun and were more motivated than those learning with numerical learning materials. Moreover, learners with high mathematics self-efficacy experienced less anxiety with regard to mathematics. Therefore, the provision of appropriate strategies to enhance mathematics self-efficacy will help learners improve their attitudes towards learning mathematics.

We suggest that teachers use strategies such as teacher feedback, goal setting, and make use of interactive models to help learners increase their mathematics self-efficacy (Siegle & McCoach, 2007).

References

Ainsworth, S. (2006). DeFT: a conceptual framework for considering learning with multiple representations. Learning and Instruction 16, 183–198.

Anjum, R. (2006). The impact of self-efficacy on mathematics achievement of primary school children. Pakistan Journal of Psychological Research, 21(3), 61-78.

Blanton, M., & Kaput, J. (2002). Developing elementary teachers' algebra eyes and ears: Understanding characteristics of professional development that promote generative and self-understanding change in teacher practice. Paper presented at the annual meeting of the American Educational Research Association, New Orleans, LA.

Bruner, J. S. (1966). Toward a theory of instruction. Cambridge, MA: Harvard University.

Dreyfus, T. & Eisenberg, T. (1996). On different facets of mathematical thinking. In R. J. Sternberg & T. Ben-Zeev (Eds.), The nature of mathematical thinking (pp.253-284). Mahwah, NJ: Erlbaum.

Fernandez, M. & Anhalt, C. (2001). Transition toward algebra. Mathematics Teaching in the Middle School, 7(4), 237-241.

Hackett, G. & Betz, N. E. (1989). An exploration of the mathematics self-efficacy mathematics performance correspondence. Journal for Research in Mathematics Education, 20(3), 261-273.

Heddens, J. W. (1984). Today,s Mathematics. (5th ed.). Chicago: Science Research Associates.

Janvier, C. (1987). Problems of Representation in the Teaching and Learning of Mathematical Problem Solving. Erlbaum, Hillsdale, NJ.

Kaput, J. J. (1987). Representation systems and mathematics. In Janvier, C. (Ed.), Problems of representation in teaching and learning of mathematics (pp. 159-195) . Hillsdale, NJ: Lawrence Erlbaum.

Lee, C. Y. & Chen, M. P. (2009). A computer game as a context for non-routine mathematical problem solving: The effects of type of question prompt and level of prior knowledge. Computers & Education , 52 (3), 530-542.

Lee, C. Y. & Cheng, C. Y. (2012). The effects of worked examples on fifth graders' flexible thinking and mathematics attitudes. Proceedings of the 2012 International Conference of Mathematics and Information Education (ICMIE2012), pp. 61-72. Taipei, Taiwan. July 16-17, 2012.

Lee, C. Y., & Chen, M. J. (2015). Effects of worked examples using manipulatives on fifth graders' learning performance and attitude toward mathematics. *Educational Technology and Society, 18*(1), 264-275.

Lee, C. Y., Chen, M. J., & Chang, W. L. (2014). The effects of multiple solution and question prompt on generalization and justification for non-routine mathematical problem solving in a computer game context. *Eurasia Journal of Mathematics, Science & Technology Education, 10*(2), 89-99.

Lee, C. Y.,& Chen, M. J. (in press). Developing a questionnaire on technology-integrated mathematics instruction - A case study of the AMA training course in Xinjiang and Taiwan. *British Journal of Educational Technology*, accepted.

Lesh, R., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier, (Ed.), Problems of representations in the teaching and learning of mathematics (pp. 33-40). Hillsdale, NJ: Lawrence Erlbaum.

Lewis, A. B. & Mayer, R. E. (1987). Students' miscomprehension of relational statements in arithmetic word problems. Journal of Educational Psychology, 79(4), 363-371.

National Council of Teachers of Mathematics (2000). The principles and standards for school mathematics. Reston, VA:NCTM.
Owen, A. (1995). In search of the unknown: A review of primary algebra. In J. Anghileri (Ed.), Children´s mathematical thinking in the primary years: Perspectives on children´s learning. London: Cassell.
Rivera, F. & Becker, J. R. (2008). Middle school children's cognitive perceptions of constructive and deconstructive generalizations involving linear figural patterns. ZDM: International Journal in Mathematics Education, 40, 65-82.
Schunk, D. H. (2007). Learning theories: An educational perceptive (5th ed.). NJ: Prentice-Hall.
Sherer, M. & Maddux J. (1982). The self-efficacy scale: Construction and validation. Psychological Reports, 51(2), 663-671.
Siegle, D., & McCoach, D. B. (2007). Increasing student mathematics self-efficacy through teacher training. Journal of Advanced Academics, 18, 278–312.
Skaalvik, E. M. & Skaalvik, S. (2006). Self-concept and self-efficacy in mathematics: Relation with mathematics motivation and achievement. Proceedings from ICLS '06 International Conference on Learning Sciences.
Tchoshanov, M. (1997). Visual mathematics. Kazan, Russia: ABAK.
Wills, G. B. & Fuson, K. C. (1988). Teaching children to use schematic drawings to solve addition and subtraction word problems. Journal of Educational Psychology, 80, 192-201.