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Teacher's Knowledge of Students about Geometry

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Abstract. This study investigated the adequacy of mathematics teachers in terms of the ability to identify students' missing knowledge and suggest strategies to address students' difficulties. The participants were 37 secondary school mathematics teachers teaching in senior classes. The teachers' years of experience range from 3-10. The teachers were requested to respond to 4 open-ended questions, and the items in the questionnaire required them to identify what knowledge the student lacked and what strategies could be used to help the student. The study revealed that most of the teachers could not indicate the student's missing knowledge with respect to angles in parallel lines. The teachers were also unable to help the student, as they could not suggest specific ways that would help remove the student's difficulties.

Keywords: Teachers' knowledge; Students' misconceptions; Angles in geometry

Introduction

Geometry as one of the branches of mathematics has an important role in the study of mathematics. Geometry is thought-about as an important branch of mathematics. According to Biber, Tuna and Korkmaz (2013), "geometry is a branch of mathematics concerned with point, straight line, plane figures, space, spatial figures, and the relations between them"(p. 1). The National Council of Teachers of Mathematics [NCTM](2000) stressed the prominence of geometry by stating that "geometry offers an aspect of mathematical thinking that is different from, but connected to, the world of numbers" (p.97). Clements and Battista (1992) pointed out that geometry can be considered as a tool to facilitate the interpretation and reflection on the physical environment. It means, through the knowledge of geometry we are able to describe, analyze and understand the world in which we live. In fact, Ozerem (2012) said, "studying geometry is an important component of learning mathematics because it allows students to analyze and interpret the world they live in as well as equip them with tools they can apply in other areas of mathematics" (p. 23). This means the understanding of the environment we live in, and the ability to do well in other areas of mathematics rest on our understanding of geometry. NCTM (1989, 2000) asserted that geometry is of benefit to both teachers and students in other topics in the mathematics curriculum and other disciplines. For instance, geometry is closely related to measurement. NCTM (2000) maintained that there is significant overlap between geometry and measurement. Problems that are related to other branches of mathematics can be solved using the knowledge of geometry, apart from its usage for solving daily life problems. Several mathematics educators have maintained that geometry promotes students' knowledge relating to space and the relationship of objects within it, skills of deductive reasoning, and the ability to solve real life problems in which geometrical vocabulary and properties present themselves (e.g. French, 2004; Presmeg, 2006; Marchis, 2012). Since the development of logical reasoning and the ability to solve real-life problems are attributable to a sound knowledge of geometry, it is necessary the teaching of geometry is done in such a manner that students' misconceptions are minimized. And this implies teachers of mathematics should be able to identify and address such misconceptions when they arise. Van Hieles (1999) pointed out that conceptual and procedural knowledge in geometry can be accelerated through instruction, and maintained that instruction is a greater determining factor of progress from one level to the next one than age or maturity.

Students' Misconceptions in Geometry

Several studies have indicated that students have problem in comprehending geometric concepts, which is an important aspect of learning mathematics (e.g. Mitchelmore, 1997; Prescott, Mitchelmore & White, 2002; Thirumurthy, 2003). Mayberry (1983) said most students learn geometry based on rote-learning approach. The student may hold the visualization and the verbal definition, but prefer the visual prototype when classifying and identifying geometric figures (Ozerem, 2012). This is indicative of rote learning. Fischbein and Nachlieli (1998) found that students were able to define parallelogram correctly, but when required to classify geometric figures according to shapes, majority of them depended on the visual prototype instead of their definitions. Researchers have given reasons for students' misconceptions in geometry. The reasons given by Ozerem (2012) include students' reliance on the physical appearances of the figures, inability to associate geometric properties with one another, overgeneralization and rote learning. Also, Clement and Battista (1992) enumerated some of the causes of students' misconceptions in geometric concepts, as (i) lack of understanding the subject sufficiently (ii) overgeneralization of specific rules (iii) rote learning and (iv) inability to comprehend geometric concepts exactly. The reasons given by Ozeren (2012) and Clement and Battista (1992) are similar, as they are centered on lack of conceptual knowledge due to rote learning approach.

Furthermore, Marchis (2012) pointed out that students have misconceptions in geometry because of concept definition. Formal concept definition generates personal concept image. Marchis (2012) asserted that this concept image may not develop in some students, and in others, it may not be related to the formal definition. Archavsky and Goldenberg (2005) found that there has often been conflict between mental images of geometric figures and verbal definitions. There is the need to address these misconceptions when teaching so that it would help the students reflect on where the confusion between the verbal definition and their own mental image comes from (Marchis, 2012). Research has shown that when classifying and identifying shapes preference is given to visual prototype rather than a formal definition (e.g. Ozerem, 2012). These misconceptions are not unconnected with the way and manner teachers handle the subject.

The literature has identified some common misconceptions in geometry among students. Mayberry (1983) and Clements and Battista (1992), said geometric shapes presented in non-standard forms are hardly recognized by many students, as they perceive a square as not a square if it is not on a horizontal base. Many students have problems in perceiving class inclusions of shapes, for example, they do not think that a square is a rectangle, or a square is a rhombus, and a rectangle is a parallelogram (Mayberry, 1983; Feza & Webb, 2005; Marchis, 2008). Other common misconceptions include, using the bottom line as the base of the triangle in calculating the area of a triangle; larger space means larger angle; inability to understand the angles in parallel lines- alternate and corresponding angles; inability to recognize and perceive the properties of quadrilaterals; learning formulas and definitions inadequately. According to Biber, Tuna and Korkmaz (2013) students lack knowledge of parallel lines and they calculate angles based on the physical appearances of the figures. In this study, the focus was on teachers' knowledge of students about angles related to parallel lines.

Though there are several studies on the investigation of students' misconceptions in geometry, the literature review indicates absence of studies on investigating teachers' knowledge of students' misconceptions in geometry, especially in Nigeria. The present study investigated the adequacy of mathematics teachers in identifying and addressing students' misconceptions in geometry, in specific, angles in parallel lines. The need for the study therefore cannot be overstressed considering the importance of geometry in school mathematics curricula and its usage for solving real-life problems.

Teachers' Knowledge of students

Educational research has identified three core components of teachers' knowledge. These are subject matter knowledge, pedagogical content knowledge, and generic pedagogical knowledge (Baumert, Kunter, Blum, Brunner, Voss, Jordan, Klusmann, Krauss, Neubrand & Tsai, 2010). The effectiveness of any teacher depends on the possession of these components of knowledge. The knowledge of concepts and procedures brought to the learning of a topic by the students, and the misconceptions the students may have developed are both aspects of the pedagogical content knowledge (Carpenter, Fennema, Peterson & Carey, 1988). This knowledge also has to do with the teachers' knowledge of methods for evaluating students' conceptual and procedural knowledge, and the adequacy of dealing with students' misconceptions.

One of the key components of teacher competence is a sound knowledge of teachers about students (Baumert et al, 2010). Teachers' knowledge of students includes ability to identify students' sources of misconceptions and to predict their thinking processes. Teachers' knowledge of students enables a variety of classroom strategies (Hill, Chin &Blazar, 2015), and ranks high among the teacher capabilities identified as important to effective teaching (Cohen, Raudenbush & Ball, 2003). Zhao (2012) pointed out that teachers' knowledge should be such that it enables students to learn skills and affect positively their learning strategies. Hill, Chin & Blazar (2008) noted that there is a general consensus among Mathematics educators that teachers who have uncommon knowledge of students' mathematical ideas and thinking are effective. And teachers who have adequate knowledge of students' mathematical ideas and thinking processes are expected to be able to identify students' difficulties in mathematics, and also the sources of their errors and misconceptions.

However, the literature reveals that mathematics teachers have difficulties in identifying students' misconceptions, and predicting students' thinking processes (Asquith, Stephens, Knoth & Alibali, 2007; Zuya, 2014). This study examined the adequacy of mathematics teachers' knowledge in identifying and dealing with students' misconceptions in geometry; specifically angles in parallel lines.

Statement of the Problem

Students' meaningful learning of geometry could help them solve and appreciate real-life problems. However, the literature reveals that students have a lot of misconceptions in learning some geometric concepts. This study was proposed to determine whether mathematics teachers are adequate in identifying and dealing with students' misconceptions associated with angles in geometry. Teachers' adequacy in this study refers to ability to identify and suggest strategies in dealing with students' misconceptions associated with angles in parallel lines.

Purpose of the Study

The main purpose of the study was to investigate mathematics teachers' knowledge of students about geometry. Specifically, the study focused on the adequacy of mathematics teachers in identifying students' misconceptions in calculating angles related to parallel lines and strategies in dealing with such misconceptions.

Research Questions

The following research questions were formulated to guide the study:

1. How adequate are mathematics teachers in identifying students' misconceptions associated with angles in parallel lines?
2. How adequate are mathematics teachers in suggesting strategies to address students' misconceptions associated with angles in parallel lines?

Methodology
Research Design
The qualitative research approach was implemented in this study. This is because qualitative method of analyzing data has emphasis on process rather than product (Woods, 2006). The focus was on how mathematics teachers explain their knowledge of students about geometry.

Participants

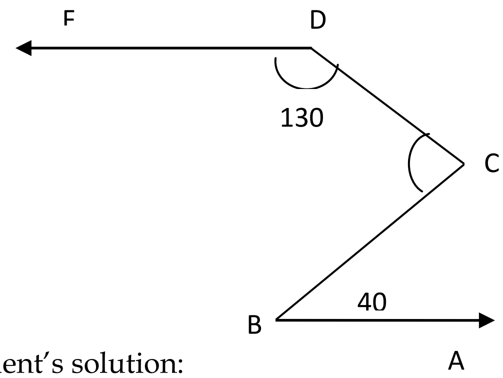
The participants were mathematics teachers randomly selected from public secondary schools in Bauchi State, of Nigeria. The participants were 37 in number, and of varying qualifications and years of experience. Their qualifications were either Nigeria Certificate in Education (NCE) or First degree in/with Education. Of the 37 participants, 12 were NCE holders and 18 degree holders. Their years of experience range from 3-10, and were teaching mathematics in the secondary schools.

Instrument for Data Collection

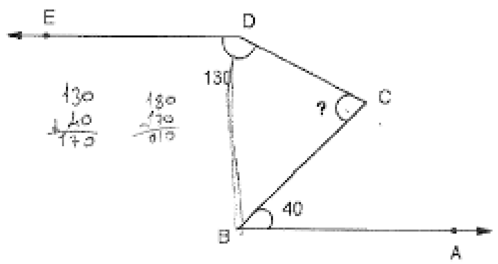
In this study, the instrument used for data collection was partly adapted from Biber, Tuna and Korkmaz (2013) and partly designed by the researcher. The misconceptions exhibited by the students in their study formed the basis for the questionnaire designed for collecting data in this study. The students' solutions were displayed, and the teachers were asked to identify the causes of the misconceptions and to suggest strategies for dealing with the misconceptions. The instrument consisted of 4 open-ended questionnaires, and each questionnaire indicates the given question and the student's solution. Questionnaires 1, 2 and 3 are shown in Figures 1, 2 and 3 respectively, and there are two items under each questionnaire. The fourth questionnaire has three items, and it is shown in Figure 4. The items designed by the researchers to collect data from the participants were given out for validation. Two experts in Mathematics Education read the items, and agreed that the items would elicit the information required.

A student was asked to calculate the angle in the figure below.

Given that DE//BA, find angle DCB.



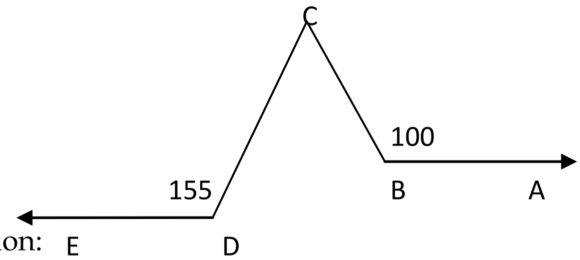
Student's solution:



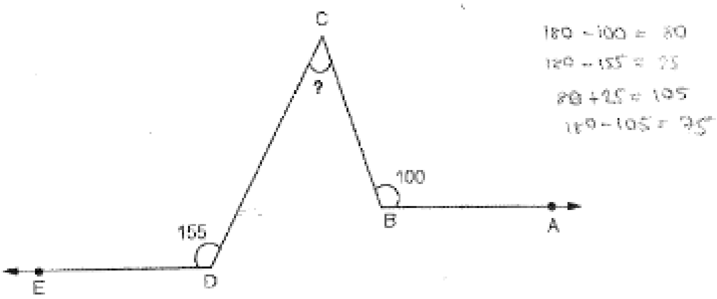
a) Identify what knowledge the student lacks.
b) How would you help this student?

Figure 1 Mathematics Teaching Questionnaire I

In the figure below, ED//BA, find angle DCB.



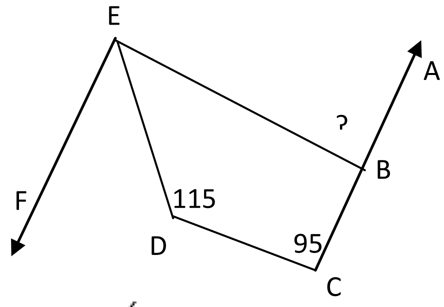
Student's solution:



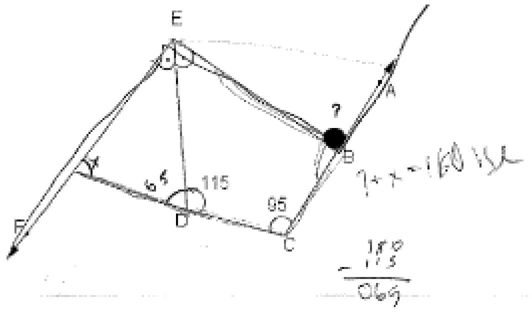
a) What does the student not know?
b) How would you help the student?

Figure 2 Mathematics Teaching Questionnaire II

Find angle EBA in the figure below, given EF//CA.



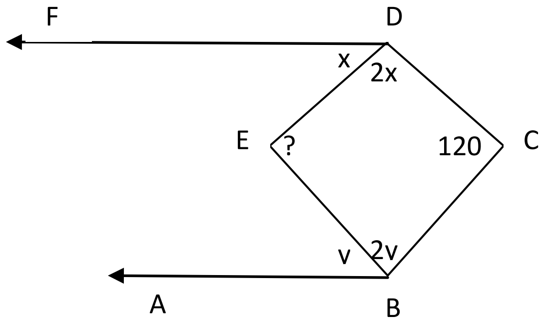
Student's solution:



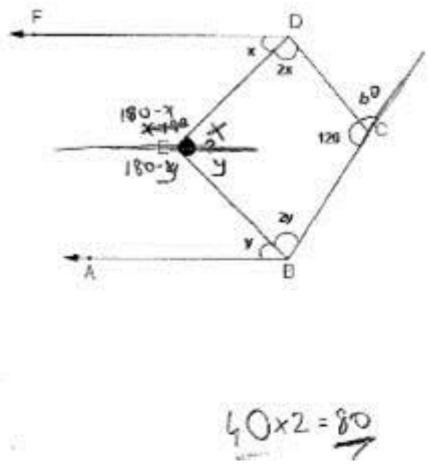
a) What knowledge does the student lack?
b) What strategies would you use to remove this misconception?

Figure 3 Mathematics Teaching Questionnaire III

Given that DF//BA, calculate angle DEB in the figure below.



Student's solution:



a) What is the student's thinking?
b) Identify what knowledge the student lacks.
c) Suggest how the misconception can be avoided.

Figure 4: Mathematics Teaching Questionnaire IV

Results and Discussion

Results

This study answered two research questions. The first research question is: How adequate are mathematics teachers in identifying students' misconceptions associated with angles in parallel lines? To answer this question, teachers were requested to respond to questions in which their answers are expected to demonstrate their ability in identifying students' misconceptions in angles. The questions required the teachers to identify students' knowledge about parallelism. Such knowledge as 'the sum of supplementary angles is 180^0', 'the sum of interior angles of a triangle is 180^0', and 'the recognition of corresponding/alternate angles as being equal'. Four different problem situations were given, and in each, teachers were requested to identify the knowledge the student lacks with respect to the displayed student's solution.

Teachers' responses to item 1(a)

Variety of responses was obtained from the mathematics teachers. Of the 37 mathematics teachers who were the respondents to the study, 5(13%) of them said "The student lacks the knowledge of angles and intercepts on parallel lines", while 5 (13%) others wrote: "The student lacks the knowledge of geometric theorems and how to apply them in solving problems". It can be said that the 5 teachers who said the student lacks knowledge of angles and intercepts on parallel lines were, to some extent, aware of the knowledge the student needed for the solution. This is because the student must know what a line which transverses two parallel lines means before he/she can make good attempt. The other 5 teachers' response was vague, as it did not identify a particular knowledge relating to some specific geometric properties.

13 (35%) mathematics teachers response was that "The student lacks the knowledge of Pythagoras' theorem". In fact, some among them said the student should have used Pythagoras' theorem in calculating the given angle. This clearly indicates inadequacy of these teachers. These teachers do not themselves have the knowledge they are expected to identify as lacking in the student. This also revealed that these teachers were relying on the physical appearance of the figure without thinking about its geometric properties.

4 (10.8%) of the teachers wrote, "The student lacks the knowledge of dividing angle C as to alternate with 40^0", 1 (2.7%) said "The student lacks knowledge of angle measurement", while 2 (5.4%) others said, "The answer to the problem should be 170^0 and not 10^0". These responses revealed that these teachers are themselves having difficulties understanding the problem situation. The response that the answer should be 170^0 and not 10^0 is irrelevant and an indication of the avoidance of the question asked.

Of the 37 teachers, 7 (18.9%) did not respond to this item. No response could mean different things. It could mean not understanding the problem situation or not having the knowledge required to solve the given problem. Whichever is applicable, there is evidence of inadequacy on the part of the teachers involved.

Teachers' responses to item 2(a)

Of the 37 teachers, 6 (16.2%) said "The student does not know that in solving or proving any geometric problem, a theorem is required to prove each step". This was in response to the question, "What knowledge does the student lack?" This response does not identify the knowledge the student lacks. The given problem situation is not on proof. The response, therefore, does not show adequacy on the part of the teachers. 17 (45.9%) other teachers stated that "the student lacks the knowledge of Pythagoras' theorem". This group of teachers either relied on the physical appearance of the geometric figure, or lacked the knowledge required to solve the problem. The knowledge required for solving the problem is the knowledge of the line that transverses two lines which are parallel, and not knowledge of Pythagoras' theorem. Still 5 (13.5%) others said "the student did not apply the rules", but did not specify which rules. And 3(8.1) of the teachers said "the student did not know that he is supposed to extend line AB to cut CD". This again is not correct identification of the knowledge the student lacks. Of the 37 teachers, 6 (16.2%) did not respond to this item. The responses of the teachers clearly indicate that they were unable to identify the knowledge required to solve the problem, as none of the responses could suggest the student's missing knowledge.

Teachers' responses to item 3(a)

On this item the teachers were expected to use their knowledge about 'parallelism', 'the sum of supplementary angles is 180^0', 'the sum of interior angles of a triangle is 180^0', or 'the sum of interior angles of a quadrilateral is 360^0'. Of the 37 teachers, 13 (35.1%) said 'the student lacks the knowledge of sum of angles in a parallelogram and alternate angles'. These teachers saw the quadrilateral as a parallelogram. 11 (29.7%) teachers responded by writing, "the student lacks the knowledge of geometric theorems". This response is too general, as it does not point to any particular theorem and there are many theorems in geometry. This is indicative of the fact that the teachers did not know which knowledge is required to solve the given problem. And 13 (35.1%) of the teachers did not respond at all to this item, which shows that the teachers themselves lack the knowledge needed to solve the problem.

Teachers' responses to item 4(a)

This item required the teachers to predict the student's thinking process. 12 (32.4%) of the teachers said "the student thinks that the figure ABCD is to be divided into two parts and extend the line to be parallel to DE". This prediction does not make sense as the line drawn by the student and the side DE touch each other. The teachers did not consider other parts of the student's solution, such as the computation. The student solution shows that parallelism was noticed, and the student wanted to apply the knowledge that alternate interior angles are equal, but unable to bring other knowledge into play. Other 8 (21.6%) teachers predicted that "the student thinks extending BC to form interior angle at C would be twice angle DEB". This prediction does not follow from the student's solution. It is not clear which angle is referred to as interior angle at C after extending BC. This is again inability to predict the student's thinking with respect to the solution. 17 (45.9%) teachers did not respond at all to this item. This indicates the teachers having difficulty themselves with the problem.

On item 4(b), teachers were requested to identify the knowledge the student lacks. Of the 37 teachers, 12(32.4%) wrote: "The student did not know that adjacent sides are equal and diagonals intersect each other at right angles". These responses are irrelevant. These teachers did not understand the problem themselves, and so could not identify what knowledge is required for solving it. The remaining 25(67.5%) teachers simply said "The student lacks the knowledge of geometry". This is vague.

Research Question 2: How adequate are mathematics teachers in suggesting strategies to address students' misconceptions associated with angles in parallel lines?

To answer this research question, teachers were requested to respond to questions that their answers would demonstrate their understanding of the subject matter and reveal their strategies in helping the students.

On problems 1 and 2, the question is: How would you help this student? Of the 37 participating teachers in this study, 18 (48.6%) responded by saying the student should be taught geometric concepts. This response is too general, and it indicates inadequacy on the part of the teachers. With respect to 1(b), one teacher said, the student should divide angle C into two so that the angle below will alternate with 40^0. This is a case of considering how the figure appears physically, and ignoring its geometric properties.

And on problems 3 and 4, the question is: What strategies would you use to remove or avoid this misconception? Of the 37 respondents, 32 (86.4%) suggested that the student should be taught. One suggested solving many similar problems as examples. In his/her words, 'I will solve many examples to show how the concepts learnt could be used in solving other problems'. It should be noted here that all those who suggested teaching as the strategy to help the student did not specify the aspect of knowledge that should be the focus of the teaching considering the problem in question. This is indicative of the inadequacy of the teachers in identifying the knowledge the student lacks.

On all the four problems, 19 (51.3%) of the teachers either attempted solving the problems or did not suggest any strategy for addressing the student's predicament. For instance on problem 1, instead of explaining how the student could be helped, a respondent tried using Pythagoras' theorem and obtained incorrect answer. This implies the teacher considered the physical appearance of the geometric figure instead of the geometric properties. In response to 'How would you help the student?' with respect to problem 2, the teacher attempted the question as 'The student should extend AB to cut CD, after extending AB, y=180-100 …' Instead of suggesting what to do to help the student, the teacher tried to solve the problem, and unfortunately could not solve it successfully.

Since the teachers were generally unable to identify the student's misconceptions or the knowledge the student lacked, they were also inadequate in addressing the student's difficulties. This has far reaching implications in the teaching and learning of geometry in particular, and mathematics in general.

Discussion

One important finding of the study was that teachers were generally inadequate in identifying the knowledge students lack with regard to angles in parallel lines. Questions 1 and 2 were very much alike; they required the student to use almost the same knowledge for solving. Teachers were unable to identify the missing knowledge because they focused on only the physical appearances of the figures. Biber, Tuna and Korkmaz (2013) working with 8^{th} grade students found that the students were at the level of visualization-focusing only on physical appearances of geometric figures. Unfortunately, this reliance on physical appearance was true of most of the teachers in this study as their responses were tailored towards physical appearances without considering the geometric properties of the figures. Majority of the teachers in this study lacked the knowledge expected of them in the subject matter. This therefore means these teachers would not be competent to teach this area of geometry. The competence of any teacher is largely dependent on the possession of the subject matter knowledge (Baumert et al, 2010). Unfortunately, these teachers did not demonstrate that they have this aspect of the teacher's knowledge from their responses.

Another important finding was the inability of the teachers to adequately suggest ways or strategies to address the student's problems. In all the four problem situations in this study, the questions were similar, requiring the teacher's knowledge of students and methods. Buamert et al (2010) pointed out that teacher's sound knowledge about students is a key component of teacher competence. Similarly, Cohen, Raudenbush and Ball (2003) said teacher's knowledge of students is necessary for effective teaching. A competent teacher must possess the necessary components of teacher's knowledge, which include subject matter knowledge, knowledge of methods and knowledge of student's cognition. None of this knowledge was demonstrated by the teachers in this study. Teacher's knowledge should help students learn skills and also enhance the ways students learn (Zhao, 2012), but regrettably these teachers did not possess such knowledge.

Conclusion

The knowledge of geometry can help appreciate the environment we live in. However, the teaching and learning of this important branch of mathematics seems to be in jeopardy, as the teachers who are expected to be knowledgeable in the area are having difficulties themselves. The failure of most of the mathematics teachers to identify the student's missing knowledge in this study calls for serious concern. As it is an indication that the teachers themselves do not possess the knowledge required to solve the problems in question. Their failure to identify the knowledge the student lacked in solving the problems in this study was not unconnected with their inability to suggest ways of helping the student. This is a case of 'you cannot give what you do not have'. Since the teachers did not have the required knowledge for solving the problems themselves, they were not adequate in pointing out the knowledge the student lacked, hence could not know what to do to help the student.

There is therefore the need to reflect on teacher education program provided by institutions concerned with the production of teachers. This is with a view to ensuring adequate preparation of teachers. Teachers' knowledge of students, which is one of the components of teachers' knowledge, is necessary for teachers' effectiveness in addressing students' difficulties.

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