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Exploring Numeracy Skills of Lower Secondary School Students in Mountainous Areas of Northern Vietnam

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Abstract. In 2018, among 10 key competencies that the Vietnam Ministry of Education and Training has proposed in the new education curriculum, numeracy was identified as an essential competency that needs to be developed for students in the future. However, little is known about the numeracy of students in general, and of students in mountainous areas in particular. The aim of this study was to design an assessment of numeracy for students in the mountainous areas to understand their performance in numeracy. The study used the quantitative approach. A sample of 730 students in grades 6-8 from eight provinces in the mountainous areas of Northern Vietnam was recruited to participate in the study. A numeracy framework and three tests with anchor items were designed and developed to measure students' numeracy. The results of the analysis using an approach to item response theory showed that the items had good fits with the model, and they could be used to describe numeracy learning progression with different levels of proficiency. The tests were reliable and valid, and the anchor items were good for connecting students' competency among grades. The results also showed that Vietnamese students in mountainous areas tended to perform better in Arithmetic and Algebra problems than in real-life problems. The results provide convincing evidence of the practical performance in numeracy of students in various ethnic minority groups in Northern Vietnam.

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1. Introduction

Numeracy is one of the important 21st century skills. When discussing the role of these skills, Steen (2007) stated that "Being numerate is one of the few essential skills that students absolutely must master, both for their own good and for the benefit of the nation's democracy and economic well-being" (p. 16). Many researchers stated that numeracy is important for individuals to develop logical thinking, reasoning strategies and problem-solving skills in their everyday activities (Westwood, 2021; Yasukawa, 2018). In many countries (Australian Curriculum Assessment and Reporting Authority, 2014; Department for Education, 2013), numeracy is an important part of the general education curriculum that needs to be developed in order to prepare students for their learning as well as their jobs in the future. It is believed that if students have poor numeracy skills, they will not only find it difficult to solve real-life problems, but also to understand materials such as news, manuals, and invoices (Thomson et al., 2020). The following sections review the definitions of numeracy, numeracy learning progression, the assessment of numeracy skills, and the context of the present research.

1.1 The Definitions of Numeracy

There are various definitions of numeracy and it has different meaning to different people (Turner, 2007). Cockcroft (1982) proposed that numeracy has two different aspects. The first aspect is people's sense of numbers and their capacity to employ mathematical skills in a way that allows them to apply these in their daily lives. The second is the ability to grasp and appreciate information from mathematical concepts. The Organisation for Economic Co-operation and Development (2012) defined numeracy as the "ability to access, use, interpret, and communicate mathematical information and ideas, to engage in and manage mathematical demands of a range of situations in adult life" (p. 33). Faragher and Brown (2005) defined numeracy as the "ability and willingness to use a wide range of mathematics in the context of people's lives" (p. 5). Westwood (2021) proposed that numeracy refers to the ability of learners to explore, understand and apply knowledge and skills in different areas such as calculation, estimation, measurement, and quantitative problem-solving. Tout (2020) also suggested that "numeracy encompasses the need for individuals to be able to understand, use and apply mathematical (and statistical) skills and knowledge" (p. 3). The author also tried to clarify the differences between numeracy skills and mathematics literacy. According to many researchers and educators, people should not consider numeracy to be equated with less mathematics and more application (O'Donoghue, 2002).

At the secondary education level, numeracy focuses more on helping students not only master mathematics knowledge and skills, especially computation skills, but also develop the competence to apply these knowledge and skills to solve real-life problems (Westwood, 2021). Moreover, students in secondary schools learn various new mathematical concepts and it is difficult for them to understand these concepts. As a result, the process of developing their numeracy may be more complicated. For instance, research on the relationships between mathematics knowledge and skills beyond school showed the existing mismatches between the two (Zawojewski & McCarthy, 2007). Various strategies and approaches have been proposed to narrow these mismatches, such as realistic mathematics education (RME) (Stemn, 2017; Venkat & Matthews, 2019).

Although there are different definitions of numeracy and many researchers have emphasised the characteristics of applying knowledge and skills in real-life situations, numeracy has different meanings in different countries. Researchers have also tried to differentiate among numeracy, quantitative literacy and mathematical literacy. These terms are still used interchangeably in some countries (Geiger et al., 2015).

1.2 Numeracy Learning Progression and Assessment

Learning progression is known as a pathway whereby teachers and students can determine students' current locations and how students can move to the next location in different domains of knowledge and skills (Heritage, 2008, 2013). Learning progression has been also described as a road map (Black et al., 2011). The main purpose of learning progression development is to support teachers in making good decisions on student learning development, as well as helping students see what they are expected to do and what they need to acquire using learning progression during their learning journey. This approach aligns with Griffin's (2018) perspectives when he proposed the developmental approach to assessment, whereby researchers need to examine a latent trait or a construct by dividing it into different dimensions, and then defining different capabilities or requirements within each dimension. After that, each capability or requirement needs to be divided into various indicative behaviours (Griffin & Care, 2015). According to McMillan (2018), one of the most important contributions when developing a learning progression is that learning progressions can overcome the disadvantages of each standard to provide stakeholders with a bigger and clearer picture about students' learning progress. Reviews from the literature showed that some countries develop numeracy learning progressions in their curricula. For instance, in the Australian national curriculum, teachers and students use learning progression as one of the sources to engage with the numeracy requirements in the Australian curriculum (ACARA, 2014). From the learning progression in the national curriculum, some states in Australia also develop their own learning progression for numeracy. For instance, in Victoria, numeracy learning progression provides a sequence of observable indicators on a scale from low to high in 15 key numeracy concepts (Department of Education and Training, n.d.).

Researchers globally have been interested in measuring numeracy. For example, Balt et al. (2020) used a development model of conceptual numerical understanding to understand numeracy learning of first-grade students. In this research, the authors used instruments to measure numeracy in different periods of time in order to analyse the growth of these skills. Strickland et al. (2016) also conducted research on measuring early numeracy skills for students with additional learning needs. They used the results from item response theory to propose a framework and a learning progression to examine students' performance on numeracy. Gittens (2015) also developed an instrument to set up a scale for measuring numeracy as an applied form of critical thinking. National assessments in some countries also measure numeracy. For instance, students in Grades 3, 5, 7 and 9 in Australia participate in the National Assessment Program – Literacy and Numeracy (NAPLAN) every year. Students' numeracy is reported against proficiency bands with ten levels. These levels show the increasing complexity of numeracy demonstrated by each student. Within each grade, students are reported using six of these ten bands (ACARA, n.d).

Apart from measuring numeracy as a whole, researchers have investigated some specific aspects of numeracy. For example, Kim et al. (2017) proposed a learning progression for geometric measurements, where they developed five levels of learning: (a) Intuitive/holistic/visual comparison; (b) Early unit concept; (c) Space filling/covering with units; (d) Interval-scale concept related to the use of efficient composite units; and (e) General model. The experts were invited to validate the learning progression, and the tasks were designed to examine students' understanding of geometric measurement in terms of length, area, and volume measurement in one, two and three dimensions. Callingham et al. (2019) developed a learning progression for the students' statistical reasoning with eight different increasing levels in statistical and probabilistic contexts. This learning progression can be used both for teaching and assessment purposes. Dole et al. (2012) and Hilton et al. (2013) also developed instruments to examine whether students could apply proportional reasoning in cross-curricular contexts. In addition, Seah and Horne (2020) developed a test to measure geometric reasoning. Moreover, many studies focus on measuring adults' numeracy in different contexts (Hoogland & Pepin, 2016; Jang et al., 2020).

1.3 The Present Research

Vietnam has developed various strategies to prepare high-quality human resources to serve the country's development in the 21st century. One of those strategies is to innovate the education system towards the development of key competencies for Vietnamese students. The Vietnam government issued a new general education curriculum in 2018. This curriculum was developed based on the competence approach in which students need to develop ten key competencies during their learning through K-12; one of these competencies is numeracy competency. Different components and detailed skills of this competency for each education level (i.e., primary, lower secondary and upper secondary level) have been defined in this curriculum (Vietnam Ministry of Education and Training, 2018). However, little is known from the literature about these competencies with the sample of Vietnamese students since this curriculum is relatively new, and the old content-based curriculum is still applicable while transforming from the old curriculum.. In the documents relating to the new general education curriculum, mathematical literacy and numeracy are sometimes used interchangeably.

In Vietnam, there are 54 minority ethnic groups of which the majority are the Kinh group with 85,4% (Open Development Vietnam, 2020), who mainly live in advantaged areas. In terms of the ethnic minority groups, the Vietnam Government has developed a wide range of policies and programmes on the education for the ethnic minority groups in order to secure equity for these groups. One such document is the Decision No. 1557/QD-TTg dated September 10, 2015 of the Prime Minister approving a number of targets for the implementation of the Millennium Development Goals for ethnic minorities in association with the goal of sustainable development after the year 2015 (Vietnam Government, 2015). This Decision also identified different strategies for the educational development of students in mountainous areas. In comparison with students of the Kinh ethnic group, these students have faced many difficulties in learning. According to a report from the Ministry of Education and Training (2015), the literacy rates for students from ethnic minority groups are lower than the national rates. Moreover, evidence from national assessments showed that the mathematics performance of ethnic minority students also tended to be lower than the national average (Vietnam Ministry of Education and Training, 2014).

Therefore, for the above reasons, there is a gap in assessing and understanding students' numeracy in the mountainous areas. Determining their learning outcomes in general, and their numeracy skills in particular is worth researching and is still relatively new in Vietnam. The aim of this study is to design a numeracy assessment for lower secondary school students in the mountainous areas in Northern Vietnam and propose a learning progression to understand the numeracy skills of these students. The present research follows the definition of Faragher and Brown (2005) that numeracy is the ability to use a wide range of mathematical knowledge and skills to solve mathematical problems as well as applying them in the context of everyday situations.

2. Methodology

2.1 Participants

A sample of 730 lower secondary school students participated in the present research. These participants are in grades 6-8 from mountainous areas in eight provinces in the north of Vietnam. In Vietnam, lower secondary school level ranges from grade 6 to grade 9. However, grade 9 students were excluded in the present research since they had to prepare for their examinations. Within each province, one school was selected by using the convenience sampling method, and within each school, 30-35 students were chosen randomly for each grade level. Table 1 shows the actual number of research participants.

Table 1. Number of Research Participants by Grade

Province	Grade 6	Grade 7	Grade 8	Total
Bac Kan	30	29	35	94
Hoa Binh	27	24	27	78
Ha Giang	29	30	35	94
Lao Cai	29	32	34	95
Lang Son	30	28	35	93
Quang Ninh	30	29	35	94
Son La	26	29	34	89
Thai Nguyen	30	28	35	93
Total	231	229	270	730

Students come from 14 of the 54 ethnic minority groups in Vietnam. The Kinh and Tay ethnic groups have the most students in the sample with 26.0% and 23.3% respectively. Among 730 students, 330 (45.2%) identified as male and 400 (54.8%) identified as female. Table 2 shows the number of students and their gender in each ethnic group within the sample.

Table 2. Number of Research Participants by Gender and Ethnic Group

Ethnic group	Male	Female	Total
Kinh	94	96	190 (26.0%)
Tay	65	105	170 (23.3%)
Dao	46	52	98 (13.4%)
Nung	41	49	90 (12.3%)
Giay	26	41	67 (9.2%)
Muong	31	28	59 (8.1%)
San Chi	11	8	19 (2.6%)
Hmong	9	8	17 (2.3%)
Cao Lan	2	4	6 (0.8%)
Thai	3	2	5 (0.7%)
Hoa	0	3	3 (0.4%)
San Chay	1	2	3 (0.4%)
Lao	0	2	2 (0.3%)
San Diu	1	0	1 (0.1%)
Total	330 (45.2%)	400 (54.8%)	730 (100%)

2.2 Research Instruments and Procedures

At the time of conducting the research, students in grades 6-8 still learned mathematics following the old curriculum. This curriculum defines different key domains of mathematics learning, including Arithmetic and Algebra, Geometry, and word problems. Although students also learn some basic knowledge of Statistics, this is not a main domain in the old curriculum. According to the new mathematics curriculum in 2018 (Vietnam Ministry of Education and Training, 2018), Probability and Statistics is considered as one of important domains. Within this domain, students start learning Probability and Statistics from Grade 2. Therefore, in the present research, all four above domains are included in the instruments to measure students' numeracy skills to connect with this new curriculum.

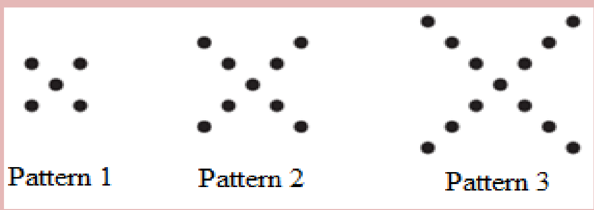
In order to explore Vietnamese students' numeracy skills, the development of numeracy learning progression was implemented. This process followed the BEAR model (Draney, 2009; Wilson, 2005). Based on the proposed definition of the numeracy skills in the previous section, a framework of numeracy skills was developed including four (4) main strands of mathematics that were defined in the curriculum. Within each strand, a set of indicators was identified to measure the competency of understanding and using relevant knowledge to solve problems within that strand of mathematics as well as in the contexts of real-life situations. In particular,, in each domain, based on the requirements in the both current and new mathematics curriculum for each grade level, a set of detailed indicators of students' numeracy skills was developed, each indicator measuring a specific aspect of numeracy skills. This set of indicators was reviewed by mathematics teachers and experts in the field of mathematics education. The full test specifications were developed for the grade 6, 7, and 8 tests. Table 3 shows an example of the item descriptions for the domain of Arithmetic in grade 6.

Table 3. Example of Item Description

Item	Item description
1	Perform arithmetical operations on the set of natural numbers
2	Recognise simple patterns of the sequence of natural numbers
3	Compare and order natural numbers
4	Generate natural numbers according to the given information
5	Represent fractions on the number line
6	Identify divisibility, division with remainder
7	Apply operation properties to solve real-life problems
8	Know how to calculate percentage

The following items measure students' skills in recognising simple patterns of the sequence of natural numbers and identifying divisibility and division with remainder:

Example Item 1. In each of the following Xs, the number of dots in each pattern is increased equally.



How many dots do you need to make the denominator 20? Show your work.

Example Item 2. Chi's father bought her a box of Vitamin C candy containing 32 tablets. Knowing that Chi eats the same amount of candy every day until the box runs out, how many candies did Ha Chi eat each day?

A. 6 tablets
B. 5 tablets
C. 4 tablets
D. 3 tablets

Table 4 introduces the number of items within each grade level for each domain. Since the grade 6 Mathematics curriculum focuses more on Arithmetic, it was proposed that most of the items should relate to Arithmetic. Regarding the domain of Statistics, each test included only two or three items since this is a main domain in the new curriculum and was not considered as a main domain in the old curriculum. In all three tests, both multiple-choice items and constructed-response items were designed for the purpose of the research.

Table 4. Items within Tests by Grade Levels and Domains

Grade	Arithmetic and Algebra	Geometry	Word problem	Statistics	Total
6	15	7	9	2	33
7	9	10	8	3	30
8	9	11	8	2	30
Total	33	28	25	7	93

In order to compare sets of all items in the same scale to develop the learning progression, a common-item nonequivalent group (CINEG) design was used (Kolen & Brennan, 2014). Numeracy tests were vertically linked across grades 6-8 by common items embedded in the tests within adjacent grade levels; that is, grade 6 and grade 7; grade 7 and grade 8. There are eight common items between each pair of adjacent grade levels. The data collection phase was implemented at the end of the school year when students had nearly finished their work for the year.

2.3 Data Analysis Procedures

Descriptive statistics were used first to have an initial understanding of the data. Then, item response theory through ConQuest (Adam et al., 2015) and R (R Core Team, 2021) software were used for the data analysis process. Specifically, since the tests consisted of both dichotomous and polytomous score items, a partial credit model (Masters, 1982) was employed to provide evidence on assessing the reliability and validity of the tests. Within the common item design among numeracy tests for grade 6-8, data for tests in each grade using item response theory was separately analysed in order to examine the quality of the common items as well as the quality of each test. The correlation analysis was also used to examine the relationships among common items within the designed tests. Then, data for all three tests was combined and the concurrent calibration equating procedures were used to set all items on the same scale (Von Davier, 2011). The information on indices of classical test theory, item difficulty, standard errors of measurement, fit statistics, reliability indices, and variable maps from the model provide evidence to assess the quality of items as well as proposing the learning progression of numeracy skills for the sample of students.

3. Results and Discussion

3.1 The quality of the linking items

As mentioned earlier, for each pair of adjacent tests, one set of item difficulties (for example, of grade 6 link items) was plotted against the other set of item difficulties (of grade 7 link items). Two scatterplots are presented below in Figure 1. In each scatterplot, each dot represents a common item. The first plot showed the relationships among all link items within two tests, and the second plot showed the relationships among link items after reviewing and selecting good link items. The fit indices and the difficulty levels of these items were also used for assessing the quality of the equating procedures (González & Wiberg, 2017). At the final stage of the equating procedures, five common items between grade 6 and grade 7 tests, and six common items between grade 7 and grade 8 tests were used for the final analysis.

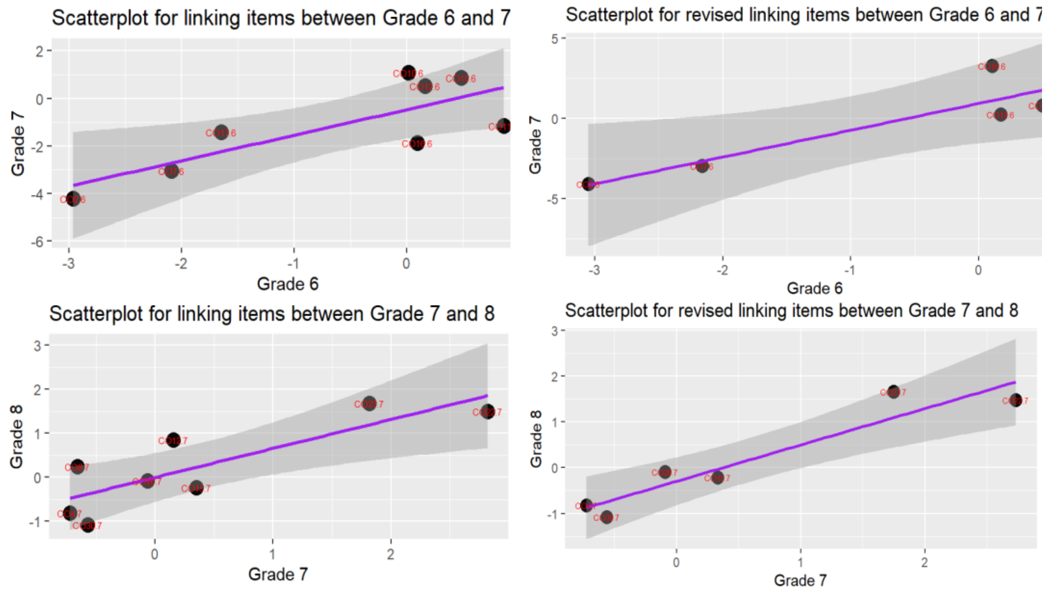


Figure 1. Relationships between Common Items within Two Tests

3.2 Reliability and Item Fit Analysis

Initial analysis showed that the set of items had a high level of internal consistency, as demonstrated by the reliability coefficient for each test (all above 0.75). According to Abu-Bader (2021), the reliability coefficients of the tests are considered to be very good. In other words, all three tests were reliable. In the first calibration phase for all 77 items of the three original tests, the estimates produced a range of 0.72 to 1.34 for the weighted mean squares (MNSQ) fit statistics. In the second calibration phase, 70 items of the three revised tests produced a range of 0.76 to 1.31. These statistics are based upon the difference between observed and expected scores and indicate how well the expected observations fit the Rasch model (Wu et al., 2016). From this result, it can be seen that all items in the revised tests had good fit indices. The final version of three tests consisted of 70 items and this version was used to run the final equating procedures and estimate item difficulties and students' abilities. All these parameters were used to propose the learning progression of numeracy skills.

3.3 Learning Progression Development for Numeracy Skills

Based on the procedure proposed by Wolfe and Smith (2007) and Wilson (2005), the standard setting process for the numeracy skills test was conducted. In this process, the information from partial credit modelling analysis (Masters, 1982) was used in connection to the work of the test designers, and the advice from secondary school teachers who were teaching Mathematics for grade 6-8 students. The set of 70 items in three tests after reviewing were used to develop a learning progression of numeracy skills which included a range from low to high difficulty levels. Following the next step in the standard setting process (Wolfe & Smith, 2007), the difficulty indices of the items in the tests were placed in order from lowest to highest, and the benchmarks were defined based on the presence of big gaps between clusters of items. Figure 2 shows the results of this process. The result of this step was to obtain categories of items along with their difficulty levels. Since each item has its description of a specific numeracy skill, within each category of items statements were written describing the main expected skills in that level.

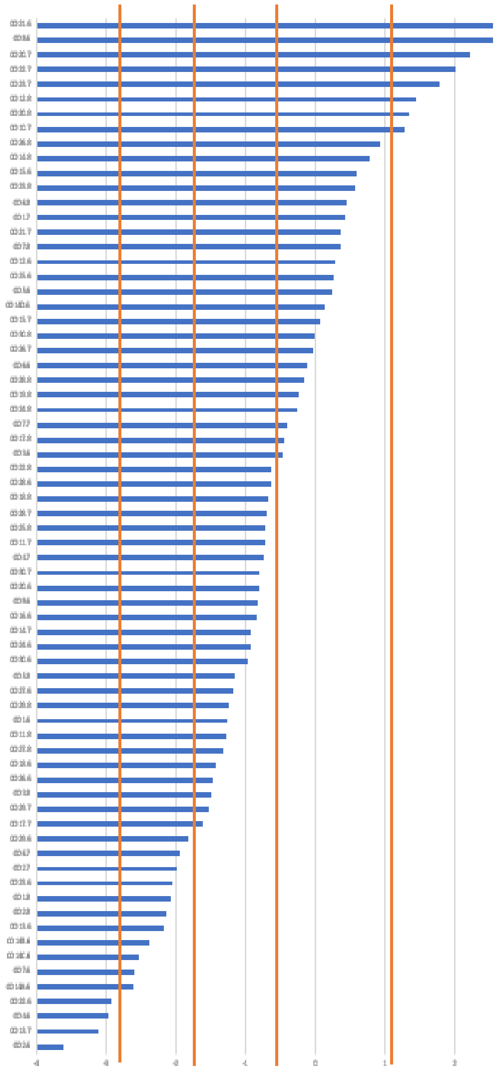


Figure 2. Item Mapping

Figure 3. ***Variable Map***

Since the results of equating procedures using item response theory put item difficulty indices and students' abilities on the same scale, it can be seen from Figure 3 that the variable map shows a good match between item difficulty and students' ability. The variable map helps stakeholders understand how the emergence of skills was developed. It also helps to interpret the results based on the learning progression. The progression derived from a review of the map is shown in Table 5.

Table 5. Derived Learning Progression of Numeracy Development

Level	Description
1	At this level, students can recognise divisibility, division with remainder, solve proportional problems, recognise types of angles, and read data from statistical tables
2	At this level, students can proficiently perform adding, subtracting, multiplying, comparing, and ordering of natural numbers; be able to perform calculations with time; recognise isosceles, equilateral, right triangles and some familiar spatial figures; can read bar charts and find the averages
3	At this level, students can make combinations of polynomials using simple operations, know how to determine whether a number is a solution to a polynomial, solve problems with time, compare decimals, calculate percentages, and recognise opposite angles, alternate angles, corresponding angles
4	At this level, students can perform calculations with polynomials, know how to represent any number on a number line, can apply their learned knowledge to solve practical situations, know how to apply Pythagorean theorem, and can calculate perimeter and area of quadrilaterals
5	At this level, students can apply knowledge of arithmetic and algebra to perform multi-step problems, solve problems of sum/difference, ratio; solve geometry problems using algebraic properties; apply knowledge of geometry such as Pythagorean theorem and knowledge of parallel lines to solve problems; can convert data in different forms and solve practical problems of related statistics to frequency and average

3.4 Vietnamese Students' Numeracy Skills

Figure 4 showed the percentages of students within each grade assigned in each level of the learning progression. It can be seen that only a few grade 6 and grade 7 students were at the lowest level of the learning progression (level 1), while there were no grade 8 students at this level. The percentages of grades 6, 7, and 8 students at the highest level (level 5) increased respectively. However, grade 6 students in the sample had a fairly high performance in compared to other grade levels since there were 57.6% of the students who belonged to level 4 of the learning progressions. There are some explanations for to this. Regarding the distribution of each domain within the Mathematics curriculum, grade 6 students focus more on the domain of Arithmetic and Algebra, with which they were fairly familiar from the previous grades. In grades 7 and 8, students have to acquire new knowledge and skills in other new domains such as Algebra, and some abstract concepts in the domain of Geometry. From a practical perspective, grade 6 students may put more effort into completing the tests. All these issues should be noted for future research.

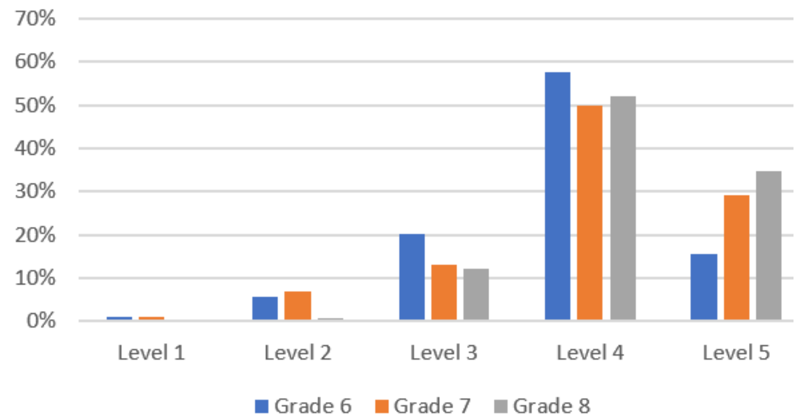


Figure 4. Percentages of Students in Each Level of the Learning Progression

In order to explore the differences in domains, Figure 6 provides a variable map from a multidimensional analysis of item response theory. Since there were only two to three items in the domain of Probability and Statistics, these items in this domain were removed from the analysis. To contribute to the previous explanation of the high performance of grade 6 students, it can be seen from Figure 5 that these students performed better in the domain of Arithmetic and Algebra when compared to the other domains, even performing better in this domain than grade 7 students.

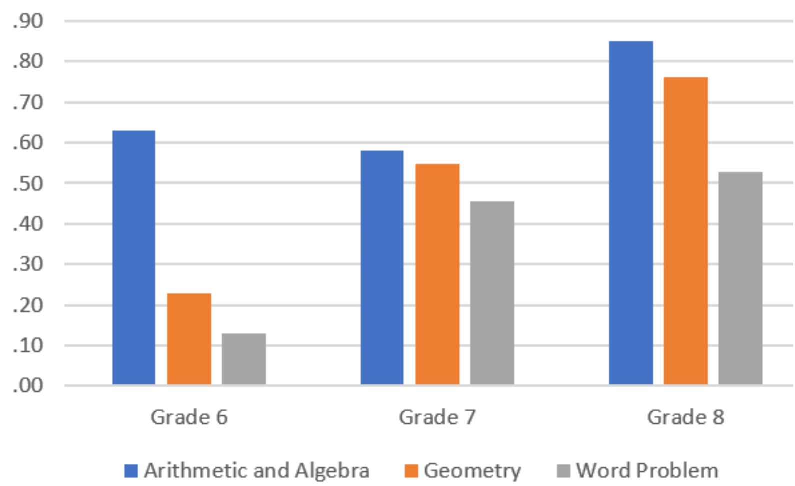


Figure 5. Logit Mean Comparison between Grades

In the domain of Geometry and word problems, there is linear growth from grade 6 to grade 8 students. From the perspectives of learning domains, one of the findings is that students in the sample had the best performance in the domain of Arithmetic and Algebra. In all three grades, students had lower performances in the domain of Geometry in comparison with the domain of Arithmetic and Algebra. Specifically, students in three grades tended to have lower performance in the domain of word problems. A word problem refers to a real-world problem where students need to use their mathematics knowledge and skills to solve it. One of the possible explanations for this finding is that at the time of the data collection phase, all students in the sample were still learning mathematics according to the old curriculum that was mostly based on mathematics content rather than the competency-based approach. This explanation is also supported by the fact that Vietnamese teachers still focus more on transmissive teaching rather than on constructive teaching (Nguyen et al., 2020). These results imply that various mathematics teaching and learning approaches should be implemented to develop students' numeracy skills though different activities.

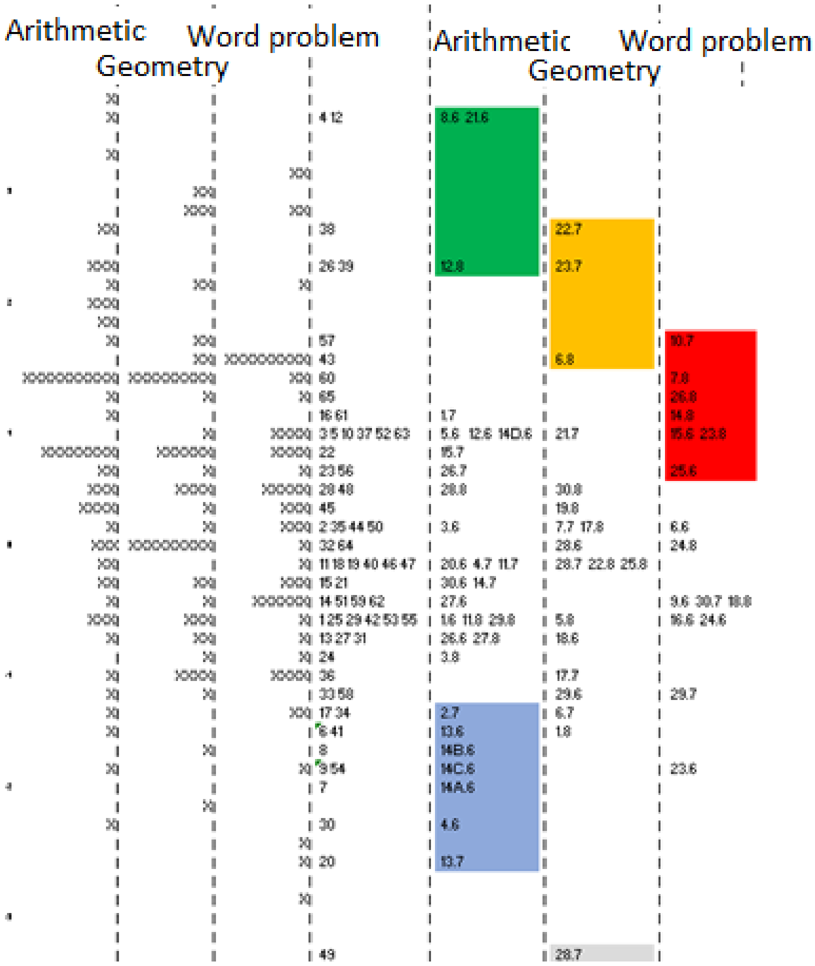


Figure 6. Variable Map for Multidimensional Analysis

4. Conclusion

The present research investigated students' performance against different levels of numeracy on a proposed learning progression. Using the support from item response theory, three tests for lower secondary school students with common items were developed and validated. Then, a learning progression of numeracy was proposed to report students' performance on numeracy in terms of different strands of mathematics. The results showed that students in the mountainous areas in the north of Vietnam performed fairly well on the tests. Specifically, students in these areas tended to perform well in the pure mathematics problems in the domain of Arithmetic, Algebra, and Geometry. Another important finding is that students are still lacking in the ability to apply mathematics knowledge and skills in everyday situations. One of the important explanations for these results comes from the fact that Vietnamese teachers are in the process of transforming from traditional teaching methods to constructive teaching approaches. The findings of the present research provide evidence of students' numeracy in mountainous areas in Northern Vietnam. From the results, there is a need for developing students' numeracy in various domains (e.g., Arithmetic, Algebra and Geometry) to increase students' performance as well as proposing mathematics learning models that focus on developing numeracy skills.

A limitation of this research is that there is no evidence of numeracy skills of students in advantaged areas to compare with the results of the study. Further research is needed to have more evidence of students' numeracy in other levels, especially of primary and upper secondary school students in other minority ethnic groups in Vietnam, and of various factors that may influence students' numeracy skills.

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