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Teachers' Professional Knowledge: The case of Variability

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Abstract. In this research, we explore teachers' statistical knowledge in relation with variability. Several high school mathematics teachers were presented with situations describing how students dealt with tasks based on the concept of variability. The teachers' responses primarily helped us to analyze their comprehension and practices associated with the concept of variability and also to gain insight on how to teach this concept. Secondly, the study shows that students and high school teachers share the same conceptions on this subject.

Keywords: teaching statistics; variability; professional knowledge; conceptions of variability.

INTRODUCTION

The importance of statistics in our lives is such that data management has become a major key in the education of responsible citizens (Konold and Higgins, 2003). The abundance of statistical data available on the internet, the studies reported on the T.V. news, or the studies and survey results published in newspapers and magazines all show that nowadays, citizens must have analytical skills in order to develop critical judgement and a personal assessment of the data they are confronted with daily.

The high amount of statistics in our society leads us to consider teaching this discipline in order to train our so-called citizens of tomorrow. If the goal is to encourage statistical thinking in students as future citizens, then not only do we need to teach basic statistical data interpretation skills, but it is also essential to teach variability. This is the foundation or statistical thinking if statistics are defined as the variability of natural and social events in our surrounding world (Wozniak, 2005).

STATISTICS, THE SCIENCE OF VARIABILITY

We live in a world characterized by variability. Let's take the example of a business that manages an urban public transit system. It may announce that its trains will arrive at the different stations every ten minutes. However, any regular transit user knows that in reality, arrival times vary and schedules are not always strictly respected. The time intervals are unequal and this lack of uniformity is the manifestation of variability. Moreover, the variable amount of travellers must also be considered. This variable reflects more or less predictable changes of schedules, seasons and the random daily variability for a given hour. In short, as shown in this example among others, variability is reflected by the absence of determinism. The complexity of the phenomena, due to the number of variables involved, is the source of this variability and of the observed variations. In the public transit example, studying the phenomenon in all its variability ensures a generally satisfying and consistent service by planning the required trains' capacity and a variable but reasonable delay between train runs.

Recognizing the variable nature of an event is also acknowledging that the results may fluctuate and be unpredictable according to sample variations. It means leaving the world of certainty and thus being able to use statistical methods to somewhat control certainty to estimate, predict and decide within and acceptable risk margin. This is often considered the main issue of statistical reasoning (Allmond and Makar, 2014; González, 2014; Vergne, 2004). A better understanding of variability helps to identify the exceptional or, conversely, to avoid false interpretations of two different results possibly based on the same law of probability. The concept of variability is also essential to hypothesis tests and statistic inference; it distinguishes statistical reasoning from reasoning associated with other areas of academic mathematics (Gattuso, 2011). Inferential statistics or statistical inference helps make general observations and draw conclusions on a given population based on random sample data collected within this same population.

The difficulty in this process is finding out to what extent the sample accurately represents the population it was collected from. In other words, how can we identify the unknown values or population parameters from the sample data? In order to illustrate this difficulty, let's imagine all possible size n samples drawn from a given population. It would be possible to calculate different varying statistics for each sample (average, variance, etc.). Therefore, any inference from a single sample necessarily comes with a probabilistic uncertainty due to sample fluctuations. The concept of variability refers to these sample fluctuations which generally decrease as the sample size increases. For example, by trying to estimate the average distance between school and the home of 30 students in a classroom from a sample of 5 people, the estimate would depend on the identity of the 5 sampled students. If 15 out of the 30 students were sampled, there would probably not be as much variation between samples as if the sample had been 5 out of the 30 possible students. In short the concept of statistical variability refers both to sample fluctuations shown in the differences between samples drawn from one population and to the statistical data dispersion which can be evaluated, among other ways, with the use of dispersion measures which show data variation in a distribution.

Based on the foregoing, it is essential to teach the concept of variability in order to develop students' statistical thinking. It is also appropriate to study teachers' knowledge of this concept as they support students and organize teaching by creating environments conducive to learning (Sánchez, da Silva and Coutinho, 2012). Nowadays, statistical training for teachers is one of the most important research fields in mathematics didactics. The ICOTS (International Conference on Teaching Statistics) is exclusively dedicated to teaching statistics. They have developed studies on the growing interest for training primary and secondary school teacher with respect to understanding the statistical concepts they teach. This purpose would allow to further develop what Skemp (1978) identifies as a relational comprehension of mathematics which can be translated into the how-to and why knowledge. These results raise important questions regarding the nature of statistical experiments which teachers may encounter during their professional training. However, before developing and offering beneficial training opportunities for teachers, it is necessary to understand how teachers comprehend statistics. Therefore, we decided to present an exploratory answer to the following question: what professional knowledge do high school mathematics teacher have about the concept of variability?

PROFESSIONAL KNOWLEDGE: FROM MATHEMATICS KNOWLEDGE BASED ON PRACTICE TO STATISTICAL KNOWLEDGE BASED ON PRACTICE.

Recent development on teachers' professional knowledge shows that this knowledge is based on the practice of teaching and is therefore related to situations from the teaching/learning context (Bednarz and Proulx, 2009; Davis and Simmt, 2006; González, 2014). Based on the works of Shulman (1988) and Ball and colleagues (Ball, Thames and Phelps, 2008; Hill, Ball and Schilling, 2008), we need to address the content and pedagogical aspects of teachers' knowledge. Content knowledge is how a specialist understands a specific field's subject matter. Pedagogical content knowledge is the ability to introduce and explain a subject going beyond content knowledge and focusing on a different dimension; understanding for teaching (Clivaz, 2014; Depaepe, Verschaffel, and Kelchtermans, 2013; Holm and Kajander, 2012; Proulx, 2008). Pedagogical content knowledge helps teachers understand what makes it easy or difficult for students to learn specific content and they rely on their own experience to help students with misconceptions and difficulties. Pedagogical content knowledge reflects the ability to organize and manage students' activities in the classroom so they may be introduced to the elements of a targeted mathematical knowledge (Bloch, 2009; Hauk, Toney, Jackson, Nair and Tsay, 2013, 2014).

In view of the above, it is possible to separate these two types of knowledge; however, in practice, they are interrelated and very hard to distinguish (Even and Tirosh, 1995). This perspective is in line with the conceptualization of professional mathematics based on the works of Moreira and David (2005), Proulx and Bednarz (2011), who differentiate academic and school mathematics as two separate knowledge fields. For example, in teaching/learning mathematical concepts, several events occur such as reasoning and understanding the concept, dealing with difficulties and errors, using problem solving strategies, encountering various representations (standardized or not) to express solutions, exploring new questions and avenues etc. These mathematical occurrences not only refer to current concepts in curricular documents, which dictate what must be taught, but also refer to mathematical elements that are part of teaching/learning mathematics which the teacher must use in class (Hauk & al., 2013, 2014). The teacher's professional mathematics knowledge refers to a body of knowledge and mathematical practices built on teaching/learning mathematics issues (Bednarz and Proulx, 2010; González, 2014). This mathematical orientation based on practice, where we don't seek to distinguish content knowledge from pedagogical content knowledge, is the essence of the present research.

METHODOLOGY

This study adopts the exploratory research focused on issues related to teaching statistics. Case studies (Yin, 2003) were developed to help answer the research question. Interviews and previously prepared questions based on specific tasks accomplishment were used as methods to collect data from teachers' answers and to better understand their ability to teach this concept. Twelve Quebec high school mathematic teachers were interviewed. The interviews were conducted at the end of the school year so these teachers had already seen a statistics module with their students. It was a two-step experiment. First, each teacher had to read an information letter inviting them to participate in the research project. It also briefly introduced the concept of variability and the purpose of the study. Introducing the concept of variability was necessary since it is not expressly defined in the Quebec school curriculum. The following definitions were presented:

-- The aim of the study is to explore how the concept of variability is taken into account while teaching.

-- The concept of variability refers, among other things, to the dispersion of data in a distribution and to sample fluctuations.

-- The possibility to quantify a distribution's variability by using dispersion measures such as the range, interquartile range and standard deviation.

This resulted in an interview where teachers were presented with cases involving the concept of variability. These consisted in analyzing the teaching curricula, reflecting on the learner's appropriation of the content by analyzing students' solutions and reasoning, and consequently to propose possible interventions to improve mathematical reasoning and understanding. These terms provided information on the teachers' professional knowledge which is directly related to mathematics teaching and learning and to their classroom practices.

As an illustration, the following two cases show examples of teachers faced with a student's answer and reasoning. These were built upon the analyses of statistical contents related to the concept of variability (didactical, conceptual and epistemological analysis; Brousseau, 1998) and inspired by analyses performed in this field (Reading and Shaughnessy, 2004; Delmas and Liu, 2005; Meletiou-Mavrotheris and Lee, 2005; Cooper and Shore, 2008).

Case example[1]

Students are each presented with a wheel as in Figure 1 below. The teacher asks them to do 5 series of 50 spins counting the number of times the arrow stops in the shaded area for each series. One student, finding this too long, decides to turn the wheel five times and multiply the result by 10 for each series. What is your opinion on this strategy and how would you respond?

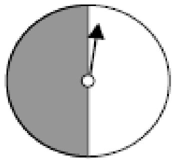

Figure1: Counting Wheel

This case shows variability in a probabilistic sampling context. Teachers were presented with a situation based on a student's misconception which was to not consider the sample size as if it had no impact on the results' variability. The student thought that the results would be the same for all 10 repetitions. By the end of high school, some students use this proportional reasoning to link samples sizes to the population's proportion (Reading and Shaughnessy, 2004). In this case, after getting 4 shaded areas after 5 rotations, a student may deduct that he or she could also get 40 shaded areas after 50 rotations. However, as the sample size increases, the features of a random sample resemble the statistical features of the population. Therefore, the variability of a size 5 sample is greater than a size 50 sample. It is also important to note that by using this strategy, it is impossible for the student to obtain the value corresponding to the theoretical probability (50% of the shaded areas). The student's approach doesn't allow for more precision because it is possible to obtain, at best, two shaded areas for three unshaded areas or vice versa.

[1] Adapted from Watson, Kelly, Callingham & Shaughnessy, 2003.

Case example 2[2]

The charts below show the height of 7th grade students from two different schools of 93 students each. Which chart shows the greatest variability in students' height?

Reasoning differently, two students offer the same conclusion: school B's chart shows a greater variability. The first student uses the fact that school B's bar chart has an oscillating pattern. The second student thinks school A's bar chart almost symmetrical and concludes that school B's chart shows greater variability. Tell us what you think of the students' answers? Which reasoning do you prefer? How would you respond to each student?

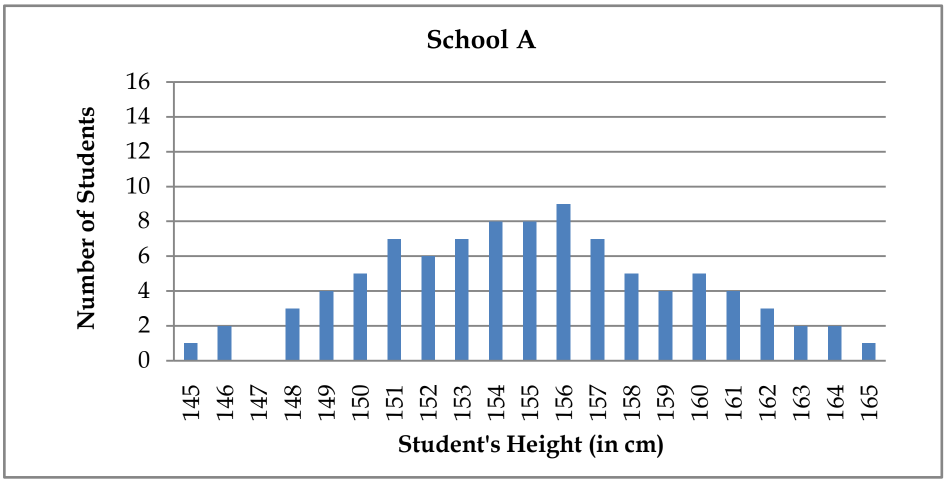

Chart 1: School A's representation of students' height.

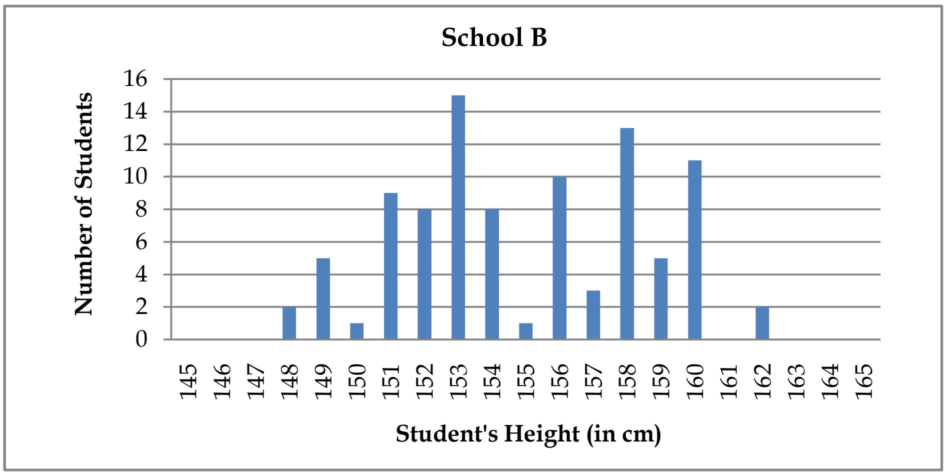

Chart 2: School B's representation of students' height.

[2] Adapted from Canada, 2004.

Here, data dispersion in both charts highlights the concept of variability. Both student's reasoning (for a task based on student's misconceptions) seen in this case are based on the works of Cooper and Shore (2008), Delmas and Liu (2005) and Meletiou-Mavrotheris and Lee (2005). These authors indicated that some students are influenced by the distribution's shape when interpreting the variability of data distribution on a bar chart or histogram. The first student was influenced by the variation in school B's bar heights. This refers more to the frequency variability rather than the variability of the subjects' height. The second student was influenced by the symmetry shown in school A's chart. The distribution's symmetry is not in indicator of variability.

Both cases aimed at seeing how the teachers dealt with the students' conception of variability in order to better understand the type of interventions they would choose. Depending on the teachers' answers, further questioning occurred during the interview in order to obtain clarifications and a deeper understanding of the professional statistical knowledge that teachers used in relation with the concept of variability. However, the interviewer's position differed from the teachers'; therefore no explanation was offered during the interview. Finally, the interviews were taped and the teachers and interviewers comments were transcribed before being analyzed. An inductive analysis process was favoured in order to identify categories from procedures identified by the researcher during the analytical process (Blais and Martineau, 2006).

RESULTS AND DISCUSSION

Two types of interventions resulted from the research; explanation and confrontation. The first one, explanation, involves reasoning to clarify concepts while answering questions. Here, students don't reflect, research or validate their knowledge. The second one, confrontation, highlights students' wrong reasoning and forces them to review and correct their conceptions. Nowadays, the cognitive conflict contribution in teaching mathematics is obvious, several authors have demonstrated the interest to challenge students' perceptions to improve their comprehension (e.g. Behr and Harel, 1990; Pratt, 1998; Steffe, 1990; Watson, 2007). The following illustrates this type of intervention base on the previous study cases.

Case example 1

Here, eight teachers responded to the issue of sample size.

Explanation

One teacher explained how the sampling size affected the sampling fluctuations. This teacher expressed knowledge related to conceptual issues and translated into an explanation to the student.

Confrontation

Two teachers proposed that in order to preserve reality, the students should experiment with different size series and compare the results. These teachers showed experimentation based knowledge by exploring the impact of the sample size on the results' variability.

Three other teachers showed students an extreme result (associating two results of zero shaded areas obtained after turning the wheel five or fifty times). One more teacher responded by exaggerating the student's reasoning. This teacher had the student use the same method but turning the wheel three times instead of five. The shaded areas' percentage will move away from the theoretical probability of 50% as the student is only able to obtain one out of two shaded area or vice-versa. One teacher suggested a different context to avoid results being transferred from a small sample to a larger one by proportional reasoning: *"By rolling a six-sided dice 5 times I would not obtain the six possible results, but if I multiply my results by 10, I could only get five different results. While this is not impossible it is highly improbable i.e. 50 rolls would produce each possible result at least once".*
To confront the student, these teachers used their knowledge of counter-examples and the variables used to build it; the results of an experiment the number of experimentations and the context.

In this situation, some teachers were not able to see the sample size issue. Of the four teachers, three accepted the student's reasoning while the fourth one refused it by pointing out that the directions had not been observed.

Case example 2

Seven out of twelve teachers responded to student's mistakes.

Explanation

Four teachers explained the problem needed to be solved horizontally and not vertically. They showed this by opposing the variability in sizes and frequencies.

Confrontation

One teacher thought the distribution shape may influence the students and so suggested to tabulate the values differently. Two teachers used counter examples suggesting a symmetrical distribution to show a low variability even though the bars varied greatly in height: *"With 14 students 153 cm tall, another 14 students 155 cm tall and 2 students 154 cm tall, the graph show a symmetrical distribution with high and low bars. Is there a big height difference? No as all students almost measure the same".*

For this task, some teachers expressed knowledge of the conceptual issue by identifying the disruptive role of the graphic aspect. This knowledge was translated into explanations to students either by an alternative presentation of the problem (transition to numbers) or by giving them a counter example.

Obviously, as it happened in the first case, some teachers were unable to identify the error, or at least recognize that the students' reasoning was mistaken. Five teachers accepted the students' wrong reasoning. One teacher valued the first reasoning more because of the greater variation of school B's bars heights. He associated the height of the bars with the height of students instead of with the frequency, thus focusing on the variability of the frequencies rather than on the variability of the variable which in this case was the students' heights. Two other teachers valued the second reasoning pretexting that school B's graphic representation looked like a bell shape associated with the regular law thus to an almost symmetrical distribution. According to them, a large variability is associated with a distribution shape which deviates from normalcy. As for the remainder, they simply noted that they couldn't disagree as they were confused by the reasoning.

It is fascinating to see the variety of interventions brought forward by the teachers and even more interesting to notice that when interpreting variability, conceptions previously observed in pupils and university students are also shared by high school teachers. In a sampling context, some teachers confirmed the use of proportional reasoning to link the sample proportions to the population proportions, as if the sample sizes did not impact the results variability, and by the same token, disregarded the samples variations.

It seems reasonable to believe that the difficulty in acknowledging variability may be due, in part, to school books which contain very few questions leading to the analysis of sample fluctuations and to the interpretation of uncertainty in favor of exercises of a more determinist nature with a focus, for example, on calculating the different statistical measures. When asked to interpret the distribution of variability from a graphic representation, some teachers were influenced by aspects associated to the distribution shape:

- Variability as a variation of the bars heights: The variation of the bar heights in a bar chart or in a histogram become an indicator of the distribution's variability; the more the bars heights vary, the greater the variability. This is a misconception of variability.
- Variability as a deviation from normalcy: The variability of a distribution is determined by its resemblance or not to the Gauss curve, the Normal; a low variability is associated to the normal shape. This is a misconception of variability.
- Variability as an asymmetrical distribution: The variability of a distribution is determined by its symmetry which in turn is associated to a low variability. This is a misconception of variability.

The resemblance in students and teachers errors is surprising. It shows a complex phenomenon related to statistics which we must understand. Common conceptions of variability seem to interfere with the notion of concept statistics. For example, it may be conceivable to associate uniformity to what varies little. This justification refers to a common language meaning and differs from the idea of statistical concept.

CONCLUSION

Although statistics is increasingly present in school programs, teacher training programs in universities give it very little attention. For example, in Quebec universities teacher training programs, no class is generally and exclusively dedicated to teaching statistics, which is not the case, notably, for geometry or algebra. This leads us to believe that mathematics teacher training regarding statistical concepts is minimal. This raises important questions on the nature of statistical experiments which teachers must perform during their professional training; the same questions apply to students. At the same time, we see a growing awareness that teachers use specific forms of knowledge, other than the standard ones learned in their university mathematics classes (Moreira and David, 2005; Proulx and Bednarz, 2010, 2011). This interaction between statistics training and classroom practice is at the core of the research project presented in this article.

This research is based on teachers' comprehension and practices in a statistical context through the exploration of various cases rooted in their practice context and by calling upon the concept of variability which is at the heart of statistical thinking. In the proposed cases, teachers were faced with students' answers and reasoning which highlighted variability related concepts. It is believed that concepts knowledge related to a particular notion helps teachers to plan their work well to organize and manage students' activity in the classroom in order for these students to learn the elements of a targeted mathematical knowledge.

Results gave way to considerations for future teacher training. Of course, the variety of interventions brought forward by these results is a starting point for ideas that could be used in class and also for future teachers training. Some interventions were proven more creative, while establishing good conditions for students to identify their errors. More importantly is the realization that some teachers could readily react to students' answers and thinking by suggesting appropriate interventions whereas others couldn't.

This context raises concerns and highlights the need to improve teaching statistics to teachers in order to continually improve their ability to intervene in the classroom in a statistical context and to develop student's statistical thinking. The interest to focus on teachers' professional knowledge in a statistical context and on the way they use it in class is even more important as we notice that, in their practice, teachers recognize more and more knowledge forms that are different from the standard ones they learned in math class in university. It is necessary to expand this knowledge so it can be remembered and used by the teachers at the appropriate moment in their practice. Research on student learning is obviously necessary for creating learning situations built on a teaching/learning context. It would allow teachers to become comfortable with how students reason in a statistical context. Teachers would learn how to intervene to improve students' reasoning and mathematical knowledge.

This mathematics orientation based on the practice of statistics is at the heart of our research on teaching statistics. It does not refer to mathematics per say which are disconnected and not set in a practice context.

REFERENCES

Allmond, S. and Makar, K. (2014). From hat plots to box plots in tinkerplots: supporting students to write conclusions which account for variability in data. *In* K. Makar, B. de Sousa and R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9)*, Flagstaff, Arizona, USA. Voorburg, The Netherlands: International Statistical Institute.

Ball, D.L., Thames, M.H. and Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59,* 389-407.

Bednarz, N. and Proulx, J. (2009). Connaissance et utilisation des mathématiques dans l'enseignement: Clarifications conceptuelles et épistémologiques. *For The Learning of Mathematics, 29*(3), 11-17.

Bednarz, N. and Proulx, J. (2010). Processus de recherche-formation et développement professionnel des enseignants de mathématiques : Exploration de mathématiques enracinées dans leurs pratiques. *Éducation et Formation*, e-293, 21-36.

Behr, M. and Harel, G. (1990). Students' Errors, Misconceptions, and Cognitive Conflict in Application of Procedures. *Focus on Learning Problems in Mathematics 12* (3-4), 75–84.

Blais, M. and Martineau, S. (2006). L'analyse inductive générale: description d'une démarche visant à donner un sens à des données brutes. *Recherches qualitatives*, *26*(2), 1-18.

Bloch, I. (2009). Les interactions mathématiques entre professeurs et élèves. Comment travailler leur pertinence en formation? *Petit x*, *81*, 25-52.

Brousseau, G. (1998). *Théorie des Situations Didactiques*, La pensée sauvage.

Canada, D. (2004). *Elementary Preservice Teachers' Conceptions of Variation.* Doctoral dissertation, Portland State University, Portland, OR.

Clivaz, S. (2014). Des mathématiques pour enseigner? Quelle influence les connaissances mathématiques des enseignants ont-elles sur leur enseignement à l'école primaire? Grenoble: La Pensée Sauvage.

Cooper, L. and Shore, F. (2008). Students' Misconceptions in Interpreting Center and Variability of Data Represented Via Histograms and Stem-And-Leaf Plots. *Journal of Statistics Education*, *15*(2), 1-13.

Davis, B. and Simmt, E. (2006). Mathematics-For-Teaching: An Ongoing Investigation of The Mathematics That Teachers (Need To) Know. *Educational Studies in Mathematics, 61*(3), 293-319.

Delmas, R. and Liu, Y. (2005). Exploring Students' Conceptions of the Standard Deviation. *Statistics Education Research Journal*, *4*(1), 55-82.

Depaepe, F., Verschaffel, L. and Kelchtermans, G. (2013). Pedagogical content knowledge: A systematic review of the way in which the concept has pervaded mathematics educational research. *Teaching and Teacher Education, 34*, 12-25.

Even, R. and Tirosh, D. (1995). Subject-Matter Knowledge and Knowledge about Students as Sources of Teacher Presentations of the Subject-Matter. *Educational Studies in Mathematics, 29*, 1-20.

Gattuso, L. (2011). L'enseignement de la statistique : où, quand, comment, pourquoi pas? *Statistique et Enseignement*, *2*(1), 5-30.

González, O. (2014). Secondary mathematics teachers' professional competencies for effective teaching of variability-related ideas : a japanese case study. *Statistique et Enseignement*, *5*(1), 31-51.

Hauk, S., Toney, A., Jackson, B., Nair, R. and Tsay, J.-J. (2013). Illustrating a theory of pedagogical content knowledge for secondary and post-secondary mathematics instruction. In S. Brown (Ed.), *Proceedings of the 16th Conference on Research in Undergraduate Mathematics Education* (Denver, CO).

Hauk, S., Toney, A., Jackson, B., Nair, R. and Tsay, J.-J. (2014). Developing a Model of Pedagogical Content Knowledge for Secondary and Post- Secondary Mathematics Instruction. *Dialogic Pedagogy*, *2*, 16-40.

Hill, H.C., Ball, D.L. and Schilling, S.G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39,* 372-400.

Holm, J. and Kajander, A. (2012). Interconnections of Knowledge and Beliefs in Teaching Mathematics. *Canadian Journal of Science, Mathematics and Technology Education*, *12*(1), 7-21.

Konold, C. and Higgins, T. (2003). Reasoning about data. *In* J. Kilpatrick, W.G. Martin and D.E. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (p. 193-215). Reston, VA: NCTM.

Meletiou-Mavrotheris, M. and Lee, C. (2005). Exploring Introductory Statistics Students' Understanding of Variation in Histograms. *Proceedings of the 4th Congress of the European Society for Research in Mathematics Education*, Sant Feliu de Guíxols, Spain.

Moreira, P. and David, M. (2005). Mathematics in teacher education versus mathematics in teaching practice. *ICMI-15 study*.

Pratt, D. (1998). The co-ordination of meanings for randomness. *For the Learning of Mathematics, 18*(3), 2–11.

Proulx, J. (2008). Exploring School Mathematics as a Source for Pedagogic Reflections in Teacher Education. *Canadian Journal of Science, Mathematics and Technology Education*, *8*(4), 331-354.

Proulx, J. and Bednarz, N. (2010). Formation mathématique des enseignants du secondaire. Partie 1 : Réflexions fondées sur une analyse des recherches. *Revista de Educação Matemática e Tecnologica Ibero-americana*, *1*(1), http://emteia.gente.eti.br/index.php/emteia.

Proulx, J. and Bednarz, N. (2011). Formation mathématique des enseignants du secondaire. Partie 2: Une entrée potentielle par les mathématiques professionnelles de l'enseignant. *Revista de Educação Matemática e Tecnologica Ibero-americana*, *1*(2), http://emteia.gente.eti.br/index.php/emteia.

Reading, C. and Shaughnessy, J.M. (2004). Reasoning about variation. *In* Ben-Zvi and J. Garfield (Eds.), *The challenge of developing statistical literacy, reasoning and thinking* (p. 201-226). Dordrecht: Kluwer Academic Publishers.

Sánchez, E., da Silva, C.B. and Coutinho, C. (2011). Teachers' understanding of variation. *In* C. Batanero, G. Burrill and C. Reading (Eds.), *Teaching Statistics in School Mathematics- Challenges for Teaching and Teacher Education*, Springer, New York, 211–221.

Shulman, L. (1988). Paradigms and research programs in the study of teaching: A contemporary perspective. *In* M. C. Whittrock (Ed.), *Handbook of research on teaching* (p. 3- 35). New York, NY: Macmillan Publishers.

Skemp, R.R. (1978). Relational Understanding and Instrumental Understanding. *The Arithmetic Teacher*, *26*(3), 9-15.

Steffe, L.P. (1990). Inconsistencies And Cognitive Conflict: A Constructivist's View. *Focus on Learning Problems in Mathematics*, *12* (3-4), 99–109.

Vergne, C. (2004). La notion de variabilité dans les programmes de seconde (2000)-Étude de conditions de viabilité didactique. Actes des XXXVIèmes journées de Statistique, Société Française de Statistique, Montpellier, France.

Watson, J.M. (2007). The Role of Cognitive Conflict in Developing Students' Understanding of Average. *Educational Studies in Mathematics, 65*, 21-47.

Watson, J.M., Kelly, B.A., Callingham, R.A. and Shaughnessy, J.M. (2003). The measurement of school students' understanding of statistical variation. *International Journal of Mathematical Education in Science and Technology*, *34*(1), 1-29.

Wozniak, F. (2005). *Conditions et contraintes de l'enseignement de la statistique en classe de seconde générale. Un repérage didactique*. Doctoral dissertation. Université Claude Bernard Lyon 1, Lyon.

Yin, R.K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks, CA: Sage.