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Chinese and American Elementary Mathematics Teachers' Between Desk Instruction

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Abstract. The purpose of this study is to understand how Chinese and American elementary mathematics teachers conduct Between Desk Instruction (BDI) and identify the similarities and differences between their actions. Qualitative research method was used to answer the research questions. During BDI, both Chinese and American elementary mathematics teachers monitored student progress, expressed inquiries that called for answers from students, provided instruction or advice at desk, guided through questioning and answered students' questions. The majority of Chinese teachers utilized BDI to select which student's work, methods or ideas needed to be shared with the whole class. All American teachers encouraged students to try their best and provided support to individual students, while only three Chinese teachers cheered students during BDI. The findings of the study contribute to our understanding of the elementary mathematics teaching practices in China and America.

Keywords: Elementary mathematics; Between Desk Instruction; Chinese teachers; American teachers.

1. Introduction

Classroom instruction plays an important role in students' mathematical thinking and learning. Teachers are key factors to classroom instruction and have an important impact on student's learning (Borko, 2016; Hiebert & Morris, 2012; Li & Kaiser, 2011; Sun & Hanna, 2013; Wagner, 2007). Ball, Lubienski, and Mewborn (2001) claimed that "What teachers and students are able to do together with mathematics in classrooms is at the heart of mathematics education" (p. 433). In order to achieve high-quality mathematics instruction in the United States, mathematics education researchers have shown great interest in investigating mathematics teachers' classroom instruction in high-achieving

countries, including China (Cai, 2000; Hino, 2006; Ma, 1999; Stigler, Lee & Stevenson, 1990; Stigler & Hiebert, 1999). Stigler and Fernandez (1995) compared the different instructional processes in American and Japanese classrooms using four categories: lesson structure, classroom discourse, role of the teacher, and pace of instruction. However, this kind of characterization

> "*can give a misleading impression that the structure of any particular lesson is independent of whether it is the introductory lesson at the commencement of a topic, a consolidation or developmental lesson later in the topic sequence, or a summative lesson occurring towards the end of a topic.*"*(*Clarke, 2004, p. 4)

Therefore, Clarke (2004) suggested using Lesson Event, which is characterized by combining variety (pictorial attributes and community contributors) and purpose (motivation, operation, inference, and consequence), as a unit of comparative analysis to analyze mathematics lessons in different countries. Clarke (2004) claimed "not only the lesson event 'Kikan- Shido' (Between Desks Instruction) (can)serve as the basis for useful comparison of classroom practice across several countries, but it also provides evidence for the co-constructed nature of a particular whole class pattern of participation" (p.7).
However, little research has been done to compare Chinese and American elementary mathematics classroom instruction, especially BDI. According to Stigler and Hiebert (2004), two goals can be achieved from the international comparative studies in mathematics: (1) to display the average mathematics classroom instruction in different countries (2) to provide mathematics teachers and researchers the opportunity to explore alternative ideas to teach mathematics. Therefore, the goal of this research is to explore how Chinese and American elementary mathematics teachers enacted BDI.

2. Theoretical basis

Classroom practice is a continued construction and interaction between teachers and students. In the following section, we are going to talk about why teachers' BDI is worthy of studying.

2.1 Why study BDI

BDI (also known as Kikan-Shido) is an activity that is familiar to teachers in every country and it serves all kinds of different functions among individual classrooms (Roche & Clarke, 2015; Shimizu, 2003). Clarke (2004) assessed the validity of BDI as a pattern of whole class participation and analyzed the activities of teacher and students with respect to this pattern of participation. Clarke et al., (2006) claimed that comparison of the enactment of BDI in different classrooms "provides significant insight into the pedagogical principles underlying the practices of different classrooms internationally" (p. 13).

Clarke, Keitel, & Shimizu (2006) analyzed eighth-grade mathematics lessons in several countries and found that the Australian mathematics teachers' BDI seemed to have at least three principal functions: monitoring and encouraging current on-task activity, scaffolding on-task activity and monitoring the

completion of homework, while German teachers normally take the action of using questions to stimulate student mathematical thinking. O'Keefe, Xu, & Clarke (2006) claimed that the activity of BDI appeared to have four mutually exclusive principal functions: (i) monitoring student activity, (ii) guiding student activity, (iii) organization of on-task activity; and, sometimes, (iv) social talk. Table 1 shows their definition of the principal functions within Kikan-Shido.

Table 1. Definition of the principal functions within Kikan-Shido (adapted from O'Keefe, Xu, & Clarke, 2006)

functions	**definitions**
Monitoring Student Activity	The process by which the teacher observes the progress of on-task activities and homework, ascertains student understanding, or selects student work, with intent to keep track of student progress, question student comprehension and record student achievement.
Guiding Student Activity	The process by which the teacher gives information, elicits student response in order to promote reflection, or facilitates engagement in classroom activity, with intent to actively scaffold the development of student participation and comprehension of subject matter.
Organizational	The process by which the teacher distributes and collects materials, or organizes the physical setting in the classroom, with intent to support interactions among students and facilitate student engagement in learning activities.
Social Talk	The teacher engages with student(s) in conversations not related to the subject matter or current on-task activity.

O'Keefe et al. (2006) further enlarged the four principal functions of Kikan-Shido into 16 activity code definitions as shown in Table 2.

Table 2. Kikan-Shido Activity Codes (O'Keefe, Xu, & Clarke, 2006)

Monitoring	M1	**Selecting Work** Students are chosen to share their work, methods or thinking with the whole class. This may occur immediately or later in the lesson.
	M2	**Monitoring Progress** Teacher walks around the classroom observing student progress of on-task activity.
	M3	**Questioning Student** An expression of inquiry that invites or calls for a reply from a student that may or may not be related to the current on-task activity.
	M4	**Monitoring Homework Completion** While students are engaged in on-task activity, the teacher observes the completion of homework and may note student achievement or understanding of subject matter.
Guiding	G1	**Encouraging Student** Activity pursued by the teacher intended to motivate, provide support and feedback to individuals or groups of students.
	G2	**Giving Instruction / Advice at Desk** Teacher scaffolds the development of students' understanding by providing information, instruction or advice, focusing on the development of a concept that addresses meaning, reasoning, relationships and connections among ideas or representations, or the demonstration of a procedure.
	G3	**Guiding Through Questioning** A series of specific teacher questions intended to scaffold the development of student understanding of a procedure or concept during the on-task activity.
	G4	**Re-directing Student** Activities pursued by the teacher to regulate the behaviour of student(s) who are perceived not to be paying attention to the current activity, and to support students' on-going engagement during the lesson.
	G5	**Answering a Question** Information given by the teacher when requested by a student.
	G6	**Giving Advice at Board** Instruction or advice given while an individual or group of students work at the board. The instruction or advice may be intended for those students working at the board or may be intended for the whole class.
	G7	**Guiding Whole Class** Teacher walks around the classroom and provides information, instruction or advice intended for the whole class.
Organizational	O1	**Handout Materials** Teacher walks around the classroom distributing materials related to on-task activity.
	O2	**Collect Materials** Teacher walks around the classroom and collects materials from students.
	O3	**Arranging Room** Teacher repositions furniture to enable independent, paired, group or board work.
Social	S1	**School Related** Teacher engages in conversation related to school activities or curriculum.
	S2	**Non-School Related** Teacher engages in conversations of a social nature not related to the subject matter or on-task activity.

The specific implementation of BDI in each classroom reflects teachers and students' participation exclusive pattern exclusive to that classroom (Clarke, Emanuelsson, Jablonka, & Mok, 2006). Although it has "universal features, its function in each classroom provides the basis for an informative comparison of those classroom practices" (Clarke, 2004, p. 2). Therefore, it is worthwhile to study the BDI to explore the similarities and differences among the teaching practice in different countries. Specifically, the research questions for this paper are: (1)What are the functions of Chinese and American elementary school mathematics teachers' BDI? (2)What are the similarities and differences between Chinese and American elementary mathematics teachers' enactment of BDI?

3. Method

In this study, qualitative research method was used to investigate how Chinese and American elementary teachers utilized BDI in their mathematics classrooms. Details about data source, coding and analysis are descried below.

3.1 Data source

The author travelled to China to collect data for the study. The participating schools included six elementary schools from an urban city in the northeast part of China and five elementary schools from the area around a major university in the south of the United States. With the help of local mathematics educators both in China and the United States, all of the volunteer participants were recruited for the study. The authors did not know any of the participating teachers. The technology teachers from each school helped the researchers video tape the lessons while the authors took notes of classroom observation. Altogether, ten elementary mathematics lessons in China and six mathematics lessons in America were videotaped. The participating teachers did not know that they were being analyzed for BDI. The lessons ranged from third grade to fifth grade. In particular, in the American schools, six different teachers (5 females and 1 male) participated in the study and in the Chinese schools, ten different teachers (9 females and 1 male) participated in the study. Unlike the American elementary teachers who were responsible for teaching not only mathematics but also language arts, science or social studies, the Chinese elementary teachers were only in charge of teaching mathematics. The average class size for the American classes and Chinese classes were 25 and 30 respectively.

3.2 Data coding and data analysis

The authors used qualitative method to examine the data by scrutinizing examples of BDI episode and discussing delicate differences among them. We used Clarke et al. (2006)'s definition of "Kikan-Shido", which is "between desks instruction in which, while the students are engaged in practice, either individually or in groups, the teacher walks around the classroom, observing students at work, and may or may not speak or otherwise interact with the students" as a guide to define BDI for this study(p. 8). For the purpose of this study, BDI is defined as the activity that teachers take while roaming around the classroom scrutinizing and interacting with students working on mathematical tasks. There are two important components in the definition. First, the definition emphasizes that BDI happens when students are working by themselves or in

groups on mathematical tasks. The second component is based on the first one. The teacher walks around the classroom and interacts with the students.
The first step in analyzing the data was to locate sessions of BDI. Each BDI session was defined by two criteria. First, it must have started from the moment the teacher told the whole class to work on their own or in small groups. Then, while students were working on the mathematical tasks, the teacher must have walked around the classroom, observing and interacting with students. BDI sessions ended when the teacher reorganized students for whole class instruction or discussion or dismissed the class.

The authors independently watched videos and marked BDI sessions from the data. Altogether, 34 sessions were identified in the data. Then we looked closely at the functions of BDI and to what extent the functions differed between Chinese mathematics lessons and American mathematics lessons. In order to do so, we coded each BDI session based on O'Keefe et al. (2006) Kikan-Shido 16 Activity Codes (see table 3). Later, both researchers met to discuss the results.
Each code was discussed between the authors until an agreement was reached.
For example, when one Chinese teacher was observing students measuring volumes of irregular objects, one group of students mentioned to her a problem they encountered while trying to measure the volume of a bottle of apple vinegar with the help of a tank of water. After she heard students' question, she paused the whole class and said, "This group of students were really good at finding problems and asking questions. Great job! They noticed that the water in their tank is not enough for them to measure the volume of the bottle of apple vinegar because the bottle of apple vinegar is not completely buried in the water. I want to ask the whole class, is it OK to measure the volume without the bottle being thoroughly covered by water?" When some students answered "no" and provided their reasoning, the teacher confirmed their opinions and reminded every group that they needed to make sure the water in their tank should be enough to cover the bottle. In this episode, the teacher took advantage of that group's question to guide the whole class that if their tank of water was not enough to cover the whole bottle, then their measurement would not lead them to the right answer. However, a closer inspection revealed that the teacher also acknowledged that group of students' effort and encouraged them to continue looking for problems and asking questions. After discussion, we agreed to code this episode encouraging students and guiding the whole class (rather than only guiding the whole class).

4. Results

We reported the findings in response to the two research questions: (a). What are the functions of Chinese and American elementary mathematics teachers' BDI? (b). What are the similarities and differences between them?

4.1 A general picture of the Chinese elementary mathematics teachers' use of BDI

A few patterns of the Chinese mathematics teachers' use of BDI were revealed. First, for the function of Monitoring Student Activity, all of the teachers walked around the classroom to observe the progress of students' on-task activity. While observing, they questioned one or two students regarding their progress, but

most importantly, they used this time to select which student's work, methods or ideas needed to be shared with the whole class. None of the teachers devoted their time to monitor students' homework completion. It might be because the whole lesson is about 40 minutes and the majority of the class time is used for students to explore the problem, explain their thinking and share with the whole class. Time for students to complete their homework was not allocated in these classes. For the function of Guiding Student Activity, most teachers encouraged students by approving their ideas or providing support and feedback, gave them advice on the development of a concept, or asked them a series of specific questions to scaffold the development of student understanding of the concept. All teachers except one utilized the common thing they observed during the BDI to guide the whole class discussion. For the Organizational function and the Social Talk, the authors did not find any from the collected videos. One of the possible reasons is that all materials that the students needed for the lesson were on students' desks before the lesson and the students were all engaged in the activities in a deliberate and positive way.

4.2 A general picture of the six American elementary mathematics teachers' use of BDI

The American teachers in this study used BDI to achieve four functions: monitoring student work, guiding student activity, organization and social work. For monitoring student activity, all of them walked around the classroom to observe students' progress on working on the mathematical task. From time to time, they expressed inquiry to individual students. All American teachers in this study reminded several students of their missing homework during BDI. For guiding student activity, American teachers always kneeled down beside a student and gave advice or instruction at desk. All American teachers in the data redirected several students' off-task behaviors during BDI. Half of the American teachers in the study handed out materials or collected materials during BDI. Two of them engaged in conversations related to school activities with students. None of them engaged in conversations unrelated to school activities with students. Detailed results of Chinese and American elementary mathematics teachers' utilization of BDI is shown in Table 3.

Table 3. Chinese and American elementary mathematics teachers' utilization of BDI

	M1	M2	M3	M4	G1	G2	G3	G4	G5	G6	G7	O1	O2	O3	S1	S2
C1	v	v	v			v	v				v					
C2	v	v	v		v	v	v				v					
C3	v	v				v			v	v	v					
C4	v	v	v		v	v	v		v		v					
C5	v	v				v	v		v	v	v					
C6	v	v	v			v			v		v					

C7	v	v				v										
C8	v	v	v			v	v			v	v					
C9	v	v	v		v	v	v		v		v					
C10	v	v	v			v	v			v	v					
A1		v	v	v	v	v		v				v			v	
A2		v	v	v	v	v	v	v	v	v			v			
A3		v	v	v	v	v	v	v	v		v	v		v	v	
A4	v	v	v	v	v	v		v		v		v				
A5		v	v	v	v	v	v	v	v				v			
A6	v	v	v	v	v	v		v			v		v			

Key: C1-C10 represents the 10 Chinese teachers;
A1-A6 represents the 6 American teachers

M1: selecting work
M2: monitoring progress
M3: questioning student
M4: monitoring homework completion
G1: encouraging student
G2: giving instruction/advice at desk
G3: guiding through questioning
G4: re-directing student
G5: answering a question
G6: giving advice at board
G7: guiding whole class
O1: handout materials
O2: collect materials
O3: arranging room
S1: school related social
S2: non-school related social

4.3 Similarities and differences

During BDI, both Chinese and American elementary mathematics teachers monitored student progress, expressed inquiries that called for answers from students, provided instruction or advice at desk, guided through questioning and answered students' questions. None of the teachers engaged in conversations not related to school activities. Most teachers in the study scaffolded the development of students' understanding through questioning. They asked a series of questions in order to elicit student thinking. They also asked students to make conjectures and justified the conjectures. The validity of the student's statement was resolved by its reasonableness and convincement of the argument, not by the wisdom of the teacher. The following quotes from one fifth grade class illustrated how the teacher used questions to guide a group of four students in their understanding of the order of making tea.
(Note: T represents the teacher. S1, S2, S3, S4 represent the four students.)

T: How are you doing? Have you all reached an agreement for the best way of making tea?
S1: No. I have a different way from his.

T: Let's take a look. (The student's order is first wash the teapot, fill water, then boil water, wash tea cups, look for tea leaves, and lastly make tea.) What do you think of it? Is it reasonable?
S2: Yes, it is.
T: Which of these steps do you think could be adjusted?
S2: look for tea leaves and wash cups could be adjusted.
T:(switched them and asked students for opinion) Are there any other things?
S3: We could also first wash the cup then wash the teapot.
T: You mean we put wash cups before wash teapot. Is that OK?
S3: Yes.
T: why?
S3: Because both washing cups and teapot are to make them clean.
T: No matter whether we wash teapot or wash cups first, it won't influence the order of making tea. Now let's take a look at which things can't be changed the order?
S1: filling water and boiling water can't be changed order.
T: Why not?
S1: because if you don't fill water, you can't boil it. You will dry the kettle.
T: what else?
S4: wash kettle and fill water
T: why not?
S4: because the reason for washing the kettle is to for sanitation.
S1:Oh, I know. So for this problem, it doesn't mean we have to have the same order of making tea. We just need to make sure the reasonableness of arranging the order.

The conversation between the teacher and the students showed that the teacher guided the students through the use of a series of questions to understand that there were many ways to arrange the order of making tea. She first made them consider whether any of these things could be adjusted and then asked them why they thought those things could be adjusted. After that, she directed them to think which could not be changed the order and why. In this way, students "construct knowledge socially, through discourse, activity, and interaction related to meaningful problems" (NCTM, 2014, p. 9).

However, distinctive differences also appeared in the video data. For example, Chinese teachers tended not to only observe student working process but also pay attention to remember which student's work needed to be picked to share with the whole class, while only two American teachers selected student work during BDI. All American teachers encouraged students to try their best and provided support to individual students, while only three Chinese teachers cheered students during BDI. Most encouragement American teachers gave to students were "Great job!" "I like you used manipulatives to solve this problem", "I think it a good idea to write an equation to represent the problem" etc. Most of these encouragements were addressed to individual students not to the whole class. On the contrary, the three Chinese teachers who provided support or encouragement to individual students or groups showed their support to individual students or groups while addressing the whole class.

Table 4 shows examples of the context and Chinese teachers' feedback to students' work during BDI.

Table 4. Encouraging Students

Context	Teacher's Encouragement
Teacher C2 asked students to work on the surface area of four cuboid stacking together. While she was observing students working individually, she noticed what Zhang Kai was doing and commented.	"Zhang Kai is really worth praising. He made a sketch of the problem, wrote down all the changes of length, width, and height, and then calculated the surface area. Very organized and clear."
Teacher C4 asked students to make a bar graph showing their classmates' birthday month. While she was walking around the classroom, she publicly cheered up an individual student named Chen Xiao.	"Chen Xiao made rapid progress recently. I saw him writing clearly, listening to the teacher's instruction carefully and solving problems correctly and quickly. Very good."
Teacher C9 provided verbal encouragement to a group of students who found a problem with their tank of water.	"This group of students was really good at finding problems and asking questions. Great job! They noticed that the water in their tank is not enough for them to measure the volume of the bottle of apple vinegar because their bottle of apple vinegar is not completely buried in the water."

5. Discussion and Conclusion

This paper attempts to describe our findings about Chinese and American elementary mathematics teachers' BDI. First, we want to talk about what we leant from the Chinese classrooms. The stereotype of Chinese mathematics classroom was the one filled with 30 to 60 students taking notes at desk and the teacher lecturing the whole time. However, Chinese mathematics classroom teaching has changed over the years since the new curriculum reform was implemented (Moy & Peverly, 2005). The new curriculum encourages teachers to use multiple teaching methods such as self-exploration, cooperation, to guide students' active learning through mathematical activities (Ding & Li, 2010; Huang, 2002; Huang & Li, 2012). The findings of this study showed that Chinese mathematics classroom instruction has altered from emphasizing rote learning, teacher-directed instruction to teacher-guided, student-centered learning, which is consistent with Moy and Peverly (2005)'s finding. Most of the Chinese lessons in the data began with teacher posing a problem to the whole class and then students either worked on their own or in small groups to solve the problem, which provided the teacher a good amount of BDI opportunity.

A distinctive feature of BDI in the Chinese lessons in the video data is the selection of student work to be shared with the whole class. Most teachers (70%) liked to let the selected students go to the board to write down their working process immediately, while other teachers (30%) waited to let the selected students shared during whole class discussion time. Usually, two or three

students' works were selected to present on the board. The whole hand-picked process was deliberate and purposeful. It represented a variety of student solution methods. At least two aims were achieved. One was that it gave voice to the student's idea and it made the student feel their thinking was valued. The other was that students got to see, compare and contrast the different ways of solving problems. This kind of action also gave shares of the responsibility for knowledge generation to the student (Ding & Li, 2014).

In harmony with the findings of O'Keefe et al. (2006), Chinese teachers usually don't encourage students during BDI, while American teachers encouraged students a lot during BDI. Encouraging students has the benefit of increasing student confidence and motivation. Students depend on teachers for validation and positive augmentation. If they feel their efforts are acknowledged and appreciated, they are more likely to be devoted to their own learning.
Another thing that we noticed from the Chinese classrooms in the study was that some Chinese teachers chose not to correct students' error immediately during BDI. They informed the students that they were wrong and asked them to either look for the mistake by themselves or discuss with their group members. If the student still could not locate the mistake, the teacher would ask the student to present the problem in front of the whole class and let the whole class explore what was wrong with the problem.

Consistent with the results of other cross-cultural comparisons of mathematics instruction in China and America (e.g., Cai, 2005; Cai, Ding & Wang, 2014; Mok, 2015), Chinese teachers paid more attention to selecting representative student work for whole class discussion, while American teachers regarded individual assistance as a priority over whole class explanation during BDI.

6. Research Limitations, Implications and Recommendations for Future Research

Even though this study illustrated the characteristics of some Chinese and American mathematics teachers' BDI, the authors are well aware of the fact that the sample size of this study was too small and the sampling procedure was very limited. The results are not generalizable to a larger population. However, the findings of this study provide an opportunity to advance understanding of mathematics classroom practices both in China and America. Teacher educators and professional development agency can also exploit the findings of this study to help student teachers or beginning teachers to improve their skills in teaching mathematics in elementary classroom (Kersting, 2008; Speer & Wagner, 2009). Future research might involve more teachers' classroom instruction and include interviewing teachers in regards to their purposes for BDI.

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