International Journal of Learning, Teaching and Educational Research
Vol. 17, No. 8, pp. 1-16, August 2018
https://doi.org/10.26803/ijlter.17.8.1

An Investigation of Pre-Service Teachers' Previous Mathematics Learning Experience from Elementary School to College and How It Relates to Attitudes and Beliefs about Mathematics Learning and Teaching

Xiongyi Liu and Roland G. Pourdavood
Cleveland State University (CSU)
Cleveland, Ohio, USA

Abstract. The purpose of the present study is to investigate pre-service teachers' previous mathematics learning experience from elementary school to college and how it relates to their attitudes and beliefs about mathematics learning and teaching. Data were collected over two semesters from a total of 67 pre-service teachers in a mathematics methods course at a mid-Western university. The results indicate that different mathematics learning tools or strategies were emphasized at different grade levels. While a few strategies that support meaningful learning showed consistent growth in their use at higher grade levels, memorization remained a heavily used strategy at all grade levels. Certain research-proven strategies (manipulatives, illustrations, measurement) were used less often at higher grade levels. Regarding the relationship between previous learning experience and current mathematics-related attitudes and beliefs, we found very limited support. Findings were discussed and educational implications were provided.

Keywords*:* Pre-service teachers; mathematics experience; attitudes; beliefs.

Introduction

Mathematics education in the United States has been criticized in recent years as student performance in mathematics continued to show more weaknesses than strengths. According to the Programme for International Student Assessment [PISA] of 2012, 15-year-old students in the United States ranked 27th in mathematics, 17th in reading, and 20th in science among the 34 member countries (Organization for Economic Cooperation and Development, OECD, 2016). U.S. students showed poor mathematics performance particularly with tasks that required them to interpret and solve real world problems (Pourdavood & Liu, 2017). When one takes into consideration the fact that the U.S. spends more per student than most countries, it is not surprising that our educational system gets a big portion of the blame.

Some researchers believe that unsatisfactory student performance in the United States and other English speaking countries may be attributed to mathematics teaching practices that emphasize rote learning, memorization, and procedural knowledge (Chen et al., 2014; Edwards, 2017; Kostos & Shin, 2010; Lee & Hannafin, 2016; McLeod, 1992; Wilcox & Monreo, 2011). However, not all researchers are willing to discredit the teaching of procedural knowledge or the use of memorization strategies. Ansari (2015) argues that mathematics teaching approach depends on the students' developmental stage. He asserts that when we ask student to reflect on their problem solving, we need to make sure that they have developed some metacognitive skills. There is some evidence that the teaching of procedural knowledge may promote the development of conceptual knowledge and memorization may have its plan in helping students acquire procedural knowledge (Rittle-Johnson, Siegler, & Alibali, 2001; Rittle-Johnson & Koedinger, 2009; Schneider, Rittle-Johnson, & Star, 2011; Fuchs et al., 2013).

While much research has been conducted regarding how different mathematics teaching approaches may provide different learning experiences for the students and hence influence their mathematics achievement and attitudes, little is known about what pre-service teachers' (PSTs') own mathematics learning experience was like or whether such experience plays a role in the development of their attitudes towards mathematics and their beliefs about mathematics teaching. In addition, it remains a debate whether memorization or meaning-making strategies should be used, to what extent, and at which grade levels. The purpose of the present study is to investigate PSTs' previous mathematics learning experience from elementary school to college and examine how it relates to their attitudes and beliefs about mathematics learning and teaching. We ask three research questions: First, is there any trend or pattern in the PSTs' self-reported experience of memorization-based vs. other learning strategies in mathematics classroom as they went through the education system? Second, is PSTs' self-reported mathematics learning experience related to their attitudes toward mathematics (interest, motivation, and confidence)? Third, is PSTs' self-reported mathematics learning experience related to their beliefs about student-centered teaching?

Literature Review

According to OECD (2016), students who reported that they used memorization strategies to learn mathematics had about the same success rate on the less challenging mathematics items as those who reported using other learning strategies. When it came to more challenging items, however, students who reported using memorization the most, were four times less likely to solve the problems than those who reported using memorization the least. There seems to be a linear, negative relationship between the use of memorization strategies and performance on advanced mathematics tasks. In addition, countries with the highest performance reported far less use of memorization than countries with less impressive mathematics performance did. Less use of memorization strategies is also associated with more positive attitudes towards mathematics including stronger interest, higher motivation, higher confidence, and lower anxiety (Wheatley, 2012; Stuart & Thurlow, 2000).

In a more recent study (Basibüyük et al., 2016), researchers analyzed 52 Turkish high school students' responses to eight questions on functions and then interviewed a sample of 13 students and 4 teachers to further explore their reasoning underneath different test responses. The results indicate that the majority of the students demonstrated inadequate conceptual understanding and experienced problems communicating their thinking and reasoning about fractions in meaningful ways. Their lack of conceptual knowledge seems to be relevant to an emphasis on memorization and operational properties in teaching.

Not all researchers would agree that mathematics teachers should not emphasize procedural knowledge or memorization strategies in their teaching. However, research (Rittle-Johnson et al., 2001; Rittle-Johnson & Koedinger, 2009; Schneider, Rittle-Johnson, & Star, 2011) indicates that conceptual knowledge and procedural knowledge are highly correlated and students learn best when their mathematics teacher alternates lessons based on each approach. Fuchs et al.'s (2013) intervention study of 385 at-risk first graders also provided evidence that following a lesson on number concepts with speeded practice of mathematics facts led to better performance than conducting the same lesson with non-speeded practice. Fuchs et al. (2013) explained that speeded practice helped at-risk children compensate for their weak reasoning ability. While affirming the benefits of memorization in mathematics learning such as reducing anxiety and enhancing fluency, OECD (2016) also states that as students get older and attend higher grade levels, the more they need to learn mathematics in a "more reflective, ambitious and creative way--one that involves exploring alternative ways of finding solutions, making connections, adopting different perspectives and looking for meaning (p. 4)".

Over the years a wide range of instructional approaches have been suggested as alternatives to the memorization and procedural approach to mathematics education. One of those alternatives is the use of *manipulatives.* Griffiths, Back, and Gifford (2017) defined manipulatives as "objects that can be handled and moved and are used to develop learners' understanding of a mathematical situation (p. 4)". Their definition includes fingers, everyday items and structured materials, with the exclusion of measuring tools, calculators, and virtual manipulatives. A comprehensive meta-analysis study by Carbonneau, Marley and Selig (2013) examined empirical evidence of the effectiveness of manipulatives for mathematics learning. Their findings indicate a small to medium overall effect of instruction using manipulatives, with relatively larger effects for retention than transfer, for children of 7-11 years old than for younger or older children. In addition, their findings suggest a small or medium effect of instruction using manipulatives for fractions and algebra than for arithmetic. Griffiths et al.'s (2017) surveyed 450 teachers whom did teach children of three to nine year olds in the U.K. and found that teachers viewed manipulatives as most appropriate for children who are younger or less competent learners. In addition, they found that teachers attributed their choice of manipulatives to many factors but they seldom chose manipulatives based on pedagogical principles. Teachers expressed a strong need for instructional guidance on how to use manipulatives in mathematics teaching and learning effectively.

The previous two decades witness the emergence of advanced *technology* tools to facilitate mathematics learning, especially graphing calculous and mobile devices such as smartphones, tablet PCs, and laptops. It is widely believed that technology use is necessary for promoting highly demanded reasoning and problem solving skills in an information society (National Council of Teachers of Mathematics (NCTM), 2015). A general consensus in previous meta-analyses (Hembree & Dessart, 1986; Ellington, 2003; Ellington, 2006; Rakes et al., 2011) is that the integration of calculators in mathematics instruction and assessment can have positive effects on students' mathematics achievement and attitudes. These meta-analyses could be biased, however, by focusing predominantly on high school students in pre-college classes. Research conducted with younger children tend to result in less conclusive findings (Vasquez & McCabe, 2002). For students with lower performance, operating the graphic calculator can be a challenging barrier itself (Drottar, 1998). Many teachers only used calculators on an irregular basis to supplement the curriculum in spite of access to them, possibly due to their preference for symbolic methods (Dewey, Singletary, & Kinzel, 2009). Furthermore, it has been reported that some students grew so dependent on the use of calculator that they were reluctant to work on mathematics problems without a calculator (Graham & Thomas, 2000). Despite these different perspectives on the use of calculators in mathematics classrooms, NCTM (2015) maintained a position supporting the use of calculators in elementary grades without replacing paper-and-pencil or mental computation methods. In addition to calculators, according to a meta-analysis by Young et al. (2012), video and computer games have been advocated for mathematics education with positive effects on student attitudes but mixed results regarding student achievement,. It is recommended that game-based learning should be situated in social interactions and game objectives should be aligned with learning objectives.

Another important aspect of promoting conceptual understanding of mathematical procedures is to integrate reading, communication, and discourse in mathematics teaching and learning. *Reading* mathematics texts is a complex process that requires a variety of skills and knowledge on the part of the student for achieving comprehension (Shuard & Rothery, 1984; Freitag, 2000). In order to fully understand a mathematics text, a reader must be able to decode and make sense of discipline-specific vocabulary, pictures, charts, graphs, symbols, notation, formulas, and equations throughout the text (Noonan, 1990). Many students struggle with mathematics reading and need much support and guidance from their teachers (Porras, 1994). On the other hand, students who develop good mathematics reading skills can benefit tremendously from engaging in problem solving on their own since mathematics texts are often written with the purpose of explaining and modeling mathematical concepts, procedures, and reasoning to the reader (Porras, 1994; Siegel et al., 1996).

The process of mathematics reading comprehension goes hand in hand with the processes of communication and discourse. The National Council of teachers of mathematics (NCTM, 2000) emphasizes that "*Communication* is an essential part of mathematics and mathematics education. It is a way of sharing ideas and clarifying understanding. Through communication, ideas become objects of reflection, refinement, discussion, and amendment (p. 60)." A key aspect of mathematics communication is writing about mathematics, which requires the use of written language of mathematics to express ideas, explain problem solving, clarify reasoning, and engage in constructive argumentation (Powell et al., 2017). Empirical evidence from vigorous intervention studies indicates that organized classroom writing can significantly improve student mathematics achievement. For example, Cohen et al. (2015) conducted a 12-week intervention with second graders and found that students who wrote about mathematics outperformed the control group in posttests of mathematical vocabulary and mathematical reasoning. Moran et al. (2014) had third graders disability students in mathematics and found significant effect of paraphrasing propositions in word problems via writing on students' problem solving performance. Iris (2009) and Cross (2009) reported similar findings with high school. In addition, Tsuruda (1994) asserts that student writing in mathematics fulfills three objectives such as student reflection, enhancing learning, and formative assessment. In spite of its effectiveness, verbal and written communication is often missing in mathematics classrooms.

The paradigm shift in mathematics education from behaviorist to social constructivism epistemology in recent years has led to an abundance of research in mathematics *discourse*. By definition, mathematical discourse refers to social interactions and communication that take place in a mathematics classroom, either between teacher and students or among students (Cobb, 1994). Research shows that both verbal and written mathematical discourse promotes conceptual understanding (Pourdavood, Wachira, & Pitre, 2015; Pourdavood & Wachira, 2015; Wachira, Pourdavood, & Skitzki, 2013). Two important aspects of classroom discourse are active listening and wait time. When a teacher asks questions, the students need to be provided adequate time to think, reflect, and respond. Research shows that the quality of student responses and overall classroom discourse are significantly improved when the teacher gives the students sufficient wait time of 3-5 seconds, even though the frequency and length of teacher input is reduced (Tobin, 1986).

Becoming mathematically literate and empowered also require competence in various aspects of *measurement*, *illustration*, and *representation*. In a modern society, a variety of measures are used ranging from common measures such as length and time to more complex measures such as humidity and population growth rates (Gravemeijer et al., 2017). Analyzing and interpreting data collected via measurement often involves the use of illustration and representation tools (e.g., words, symbols, maps, functions, tables, graphs, charts, etc.) for understanding the relevant mathematical dimensions, shapes, patterns, relationships, and probabilities. Visualization is an integral component of illustration and representation. Arcavi (2003) defined visualization as "the ability, the process and the product of creation, interpretation, use of and reflection upon pictures, images, diagrams, in our minds, on paper or with technological tools, with the purpose of depicting and communicating information, thinking about and developing previously unknown ideas and advancing understandings (p. 217)". The value of visualization in learning and doing mathematics has been long recognized. It is considered a prominent tool to illustrate mathematical concepts, explore mathematical relationships, and solve mathematical problems (Rau, Aleven, & Rummel, 2015).

Method

A total of 67 out of 70 pre-service teachers returned completed questionnaire (see appendix 1). The participants were recruited from three sections of a mathematics methods course at a Mid-western university during Fall 2016 and Spring 2017. The course has a dual numbered section that includes both graduate students and undergraduate students. Out of 67 participants, 64 were undergraduates and three were graduates. Ten out of 67 students were special education major with emphasis on mild/moderate intervention specialist program and 57 were in early childhood program. All but eight of the participants were doing their practicum in urban and suburban pre-k-3rd grades settings during the course of the study. The participants were mostly female (88.1%), and based on self-report of 61 out of 67, they had an average age of 24.42 years old.

The course instructor distributed the questionnaires in the classroom during the middle of the semester and asked the students to bring them back in a week. Participation was voluntary and anonymous. The questionnaire consists of demographic questions about the student's gender and age, a scale that measures their previous mathematics learning experience, a mathematics attitude scale with three single items measuring mathematics interest, mathematics motivation and mathematics confidence, and a scale on beliefs about student-centered teaching approaches (see appendix 1).

Mathematics Learning Experience Scale

The authors developed this scale to measure PSTs' previous experience of mathematics learning. There are nine subscales, each focusing on a specific type of mathematics learning experience: memorization, technology, measurement, manipulatives, multiple representation, reading, discourse, illustration, and communication. Each subscale includes five statements covering five levels of education: elementary school, middle school, high school, college, and adult workplace. For example, the first item in the subscale Memorization is "In elementary school, I memorized facts, rules or algorithms." Students are asked to rate the frequency that they engaged or engage in each type of math learning experience, using a five-point scale (1 = never, 2 = rarely, 3 = sometimes, 4 = often, and 5 = always). For the purpose of the present study, we chose not to use data on the fifth item in each subscale since those items focus on expectation for future workplace experience, which is not relevant to the research questions of the present study.

Beliefs about Student-Centered Teaching

This scale was developed by the authors based on qualitative analysis of PSTs' reflection during practicum and internship. It uses six items to measure PSTs' beliefs about different student-centered approaches in math teaching. An example item is "Students should view their teacher as a facilitator of learning rather than the dispenser of knowledge." The rating scale is a five-point Likert-type scale that goes from 1 (strongly disagree) to 5 (strongly agree). There is some evidence of reliability of the scale, with a Cronbach's alpha of .729.

Results

As indicated in Figure 1 below, significant changes occurred as students went to higher grade levels. The trend, however, differs for each specific type of mathematics learning experience. For example, regarding the use of discourse, reading, and communication as mathematics learning strategies, there seems to be a positive, linear relationship. As students went to higher grade levels, they reported increasing use of those strategies.

Students' technology use in mathematics learning consistently increased from elementary to high school though it remained at the same level from high school to college. Regarding the use of illustration and manipulatives, the data showed a gradual decrease from elementary school to high school but not much change between high school and college. Regarding the use of measurement, there was not much change from elementary school to middle school but then it went down gradually during high school and in college. The use of memorization and multiple representations in mathematics learning showed an interesting, quadratic pattern across grade levels, being lower in elementary school and in college but higher in middle school and high school.

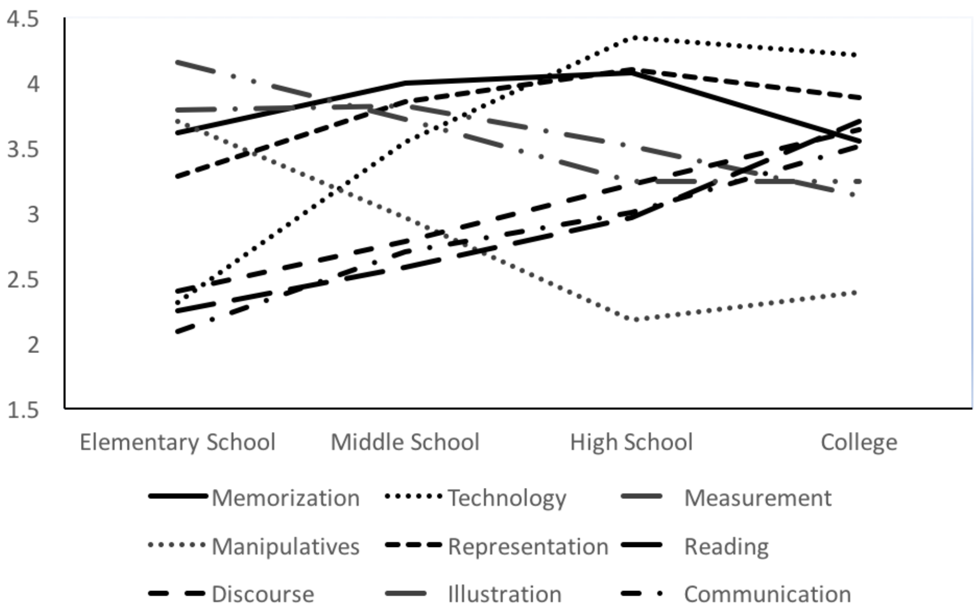

Figure 1. Engagement in Nine Types of Math Learning Experience during Elementary School, Middle School, High School, and College

We examined pairwise correlations between the nine aspects of participants' mathematics learning experience and their current attitudes and beliefs about mathematics. Specifically, we included mathematics interest, mathematics motivation, mathematics confidence, and student-centered teaching belief in our analysis. As shown in Table 1, mathematics interest, motivation and confidence were highly correlated with each other (r = .74, .75, and .81, p <.01) though none of them was correlated with student-centered teaching belief.

Table 1. Correlation Coefficients among Mathematics Attitudes, Mathematics Teaching Beliefs, and Previous Mathematics Learning Experience

	1	2	3	4	5	6	7	8	9	10	11	12	13
1. Interest	1												
2. Motivation	.81**	1											
3. Confidence	.75**	.74**	1										
4. SCT	.17	.03	-.06	1									
5. Memorization	.22	.13	.23	.12	1								
6. Technology	-.04	-.06	-.14	.11	.11	1							
7. Measurement	.02	-.10	-.07	.06	.11	.22	1						
8. Manipulatives	-.01	.06	.09	-.06	.08	.24	.30*	1					
9. Representation	.19	.09	**.30***	.05	.34**	.15	.36**	.39**	1				
10. Reading	.15	.17	.20	-.08	.30*	.11	.30*	.45**	.33**	1			
11. Discourse	**.32****	**.29***	.09	.06	.14	.16	.23	.28*	.30*	.60**	1		
12. Illustration	.08	-.02	.06	-.00	.18	.19	.31*	.46**	.52**	.34**	.27*	1	
13. Communication	.09	.05	.20	-.11	.22	-.07	.26*	.29*	.42**	.62**	.50**	.45**	1

Notes. SCT = Student-Centered Teaching. * p < .05. ** p < .01.

Among the nine types of math learning experience, measurement, manipulation, multiple representation, reading, discourse, illustration, and communication were all significantly correlated with each other. Memorization was only significantly correlated with multiple representation (r = .34, p < .01) and reading (r = .30, p < .05) whereas technology use was not correlated with any other variable.

Next, we examined the relationship between previous mathematics learning experience and participants' current attitudes and beliefs. By and large, there were few significant correlations. None of the nine indicators of previous mathematics learning experience was related to student-centered teaching belief. Use of multiple representations was significantly related to mathematics confidence (r = .30, p <.05). Use of discourse was significantly related to mathematics interest (r = .32, p < .01) and mathematics motivation (r = .29, p < .05). Since there were few significant correlations and even the three significant correlations were marginally medium sized, we decided not to proceed with any regression analysis.

Conclusion

In the current study, we examined the pattern of pre-service teachers' mathematics learning experience from elementary school to college. The results indicate that different mathematics learning tools or strategies were emphasized at different grade levels. The PSTs reported steadily increasing use of discourse, reading and communication as they went from elementary school to college.

However, such an experience was not accompanied by decreasing usage of memorization strategies. In fact, memorization stayed as a popular learning strategy throughout the PSTs' school life and even showed slightly elevated use during high school and in college. A similar curvilinear pattern was found regarding the use of multiple representations. The use of technology showed the largest gain from elementary school to high school. Finally, the use of manipulatives and illustrations showed a gradual decline from elementary to high school, while the use of measurement kept declining from elementary school to college.

The pattern reported above regarding the PSTs' mathematics learning experience asks for a more sophisticated understanding of effective use of mathematics learning strategies at different educational stages. While it is encouraging to see that the strategies that have more potential for promoting meaningful learning were experienced more and more by the PSTs as they went to higher level of schooling, it is somewhat disturbing to see that memorization continued to be widely used regardless of the grade level. At the same time, how should we explain the gradual declining use of manipulatives and illustrations from elementary school to high school? Were manipulatives and illustrations considered less useful for mathematics learning by middle school teachers than by elementary school teachers, and even less so by high school and college teachers? Why was measurement used to the same extent during elementary school and middle school, but to a less extent by high school and even less in college?

We highly recommend the use of a variety of tools and strategies to facilitate meaningful learning of mathematical concepts and ideas at all educational levels. It will be worthwhile to explore the reasons behind middle school and high school teachers' growing reluctance to the use of manipulatives, illustrations, and measurement in mathematics teaching and learning. Once we have a better understanding of the reasons, more efforts would be needed to help these teachers overcome barriers to effective use of such strategies. One way to support these teachers is to help them integrate technology into some relatively "old-fashioned" strategies (Bahng & Lee, 2017; Dietze & Kashin, 2013). For example, research shows that virtual manipulatives can be as effective as physical manipulatives (Satsangi et al, 2016; Moyer-Packenham et al., 2014). Other suggestions (Callaghan et al., 2017; Craig, 2000; Edwards, 2017) include the integration of collaborative learning, games, language art, interdisciplinary learning, complex problem solving, real life connections, etc. Finally, these tools and strategies can be combined in creative ways to make learning more engaging and productive. It is not necessary to view them as parallel strategies. For example, students can share their illustrations of the same mathematical concept and then engage in critical discourse about them.

Our findings regarding the relationship between the PSTs' previous mathematics learning experience and their current mathematics attitudes and beliefs indicate that for most of the learning tools or strategies of interest, frequency of use was unrelated to either attitudes towards mathematics or beliefs about mathematics teaching. Discourse and multiple representations were the only two exceptions, with discourse positively related to mathematics interest and motivation and multiple representations positively related to mathematics confidence. The correlations, though significant, were all around .30, barely meeting the cut-off for medium correlation coefficients. On the one hand, such findings suggest that we should continue to advocate the use of discourse and multiple representations in mathematics education. On the other hand, both teachers and students need not only more tools, but also more consistent and positive experience with the use of tools and strategies that support meaningful learning. Even though the PSTs generally believed in student-centered teaching, such beliefs did not seem to come from their own learning experience as a K-12 or even college student taking mathematics courses. Future studies should explore teachers' experience with incorporating specific strategies and how students receive them.

Acknowledgment:
A short draft of this research paper was presented at the *proceedings of 14th International Conference of the Mathematics Education for the Future project: Challenges in mathematics Education for the Next decade,* September 10-15. 2017, Balatonfüred, Hungary.

Endnote:
We obtained the *Cleveland State University Institutional Review Board*'s (CSU, IRB) approval for conducting this research study. This paper has not been previously published, nor is it before another journal for consideration.

About the Authors:
Xiongyi Liu is an associate professor of educational psychology at Cleveland State University, Department of Curriculum and Foundations. Her research interests include motivation, self-regulation, collaborative learning, and peer assessment. Her research has a special focus on computer based learning environments.

Roland Pourdavood is a professor of mathematics education at Cleveland State University, Department of Teacher Education. His research interests include mathematics teachers' dialogue and reflection for transformation and school reform. In addition, he focuses on cultural diversity, sociocultural aspects of education, and emancipatory action research for personal and social praxis.

References

Ansari, D. (2015, Sept.). No more math wars: An evidence-based, developmental perspective on math education. *Education Canada, 55*(3). Retrieved from https://www.edcan.ca/articles/no-more-math-wars/

Arcavi, A. (2003). The role of visual representations in the learning of mathematics. *Educational Studies in Mathematics, 52*(3), 215-241. doi:10.1023/A:1024312321077

Bahng, E. J., & Lee, M. (2017). Learning experiences and practices of elementary teacher candidates on the use of emerging technology: A grounded theory approach.

International Electronic Journal of Elementary Education, 10(2), 225-241. doi:10.26822/iejee.2017236118
Basibüyük, K., Sahin, Ö., Gökkurt, B., Erdem, E., & Soylu, Y. (2016). The mistakes that are made by students with regard to functions: Evidence from Erzincan (a province in Turkey). *Universal Journal of Educational Research, 4*(11), 2523-2532. doi:10.13189/ujer.2016.041105
Callaghan, M. N., Long, J. J., van Es, E. A., Reich, S. M., & Rutherford, T. (2017). How teacher integrate a math computer game: Professional development use, teaching practices, and student achievement. *Journal of Computer Assisted Learning, 34*(1), 10-19. doi:10.1111/jcal.12209
Carbonneau, K. J., Marley, S. C., & Selig, J. P. (2013). A meta-analysis of efficacy of teaching mathematics with concrete manipulatives. Journal of Educational Psychology, 105(2), 380-400. doi: 10.1037/a0031084
Craig, D. V. (2000). Technology, math, and the early learner: Models for learning. *Early Childhood Education Journal, 27*(3), 179-184. doi:10.1007/bf02694232
Chen, J., McCray, J., Adams, M., & Leow, C. (2014). A survey study of early childhood teachers' beliefs and confidence about teaching early math. *Early Childhood Education Journal, 42*(6), 367-377. doi:10.1007/s10643-013-0619-0
Cobb, P. (1994, Oct.). Where is mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher, 23,* 13-20. doi: 10.2307/1176934
Cohen, J. A., Miller, H. C., Casa, T. M., & Firmender, J. M. (2015). Characteristics of second graders' mathematical writing. *School Science and Mathematics, 115*, 344−355. doi:10.1111/ssm.12138
Cross, D. I. (2009). Creating optimal mathematics learning environments: Combining argumentation and writing to enhance achievement. *International Journal of Science and Mathematics Education, 7*, 905−930. doi:10.1007/s10763-008-9144-9
Dewey, B., Singletary, T. J., & Kinzel, M. T. (2009). Graphing calculator use in Algebra teaching. *Graphing Calculators, 109*(7), 383-393. doi:10.1111/j.1949-8594.2009.tb17869.x
Dietze, B., & Kashin, D. (2013). Shifting views: Exploring the potential for technology integration in early childhood education programs. *Canadian Journal of Learning and Technology, 39*(4), 1-12. doi:10.21432/t25p4z
Drottar J. F. (1998). An analysis of the effect of the graphing calculator on student performance in algebra II. *Dissertation Abstracts International, 60*(1), 56A. (UMI No. 9915557)
Edwards, S. (2017). Play-based learning and intentional teaching: Forever different. *Australian Journal of Early Childhood, 42*(2), 4-11. doi:10.23965/ajec.42.2.01
Ellington, A. J. (2003). A meta-analysis of the effects of calculators on students in precollege mathematics classes. *Journal for Research in Mathematics Education, 34,* 433–463. doi:10.2307/30034795
Ellington, A. J. (2006). The effects of non-CAS graphing calculators on student achievement and attitude levels in mathematics: A meta-analysis. *International Journal of Instructional Media, 106*, 16–26. doi:10.1111/j.1949-8594.2006.tb18067.x
Freitag, M. (2000). Reading and writing in the mathematics classroom. *The Mathematics Educator, 8*(1), 24-29.
Fuchs, L. S., Geary, D. C., Compton, D. L., Fuchs, D., Schatschneider, C., Hamlett, C. L. Seethaler, P. M., Wilson, J., Craddock, C. F., Bryant, J. D., Luther, K., & Changas, P. (2013). Effects of first-grade number knowledge tutoring with contrasting forms of practice. *Developmental Psychology, 105*, 58-77. doi:10.1037/a0030127
Graham, A. T., & Thomas, M. O. J. (2000). Building a versatile understanding of algebraic variables with a graphic calculator. *Educational Studies in Mathematics, 41*(3), 265-282.

Gravemeijer, K., Stephan, M., Julie, C., Lin, F.-L., & Ohtani, M. (2017). What Mathematics Education May Prepare Students for the Society of the Future? *International Journal of Science and Mathematics Education, 15*(Supp.1), 105-123. doi:10.1007/s10763-017-9814-6

Griffiths, R., Back, J., & Gifford, S. (2017). *Main Report-Using manipulatives in the foundations of arithmetic.* Retrieved from the Nuffield Foundation website: https://www.nuffieldfoundation.org/sites/default/files/files/Nuffield%20Main%20Report%20Mar%202017web(1).pdf

Hembree, R., & Dessart, D. (1986). Effects of hand-held calculators in precollege mathematics education: A meta-analysis. *Journal for Research in Mathematics Education, 17*, 83–99. doi:10.2307/749255

Idris, N. (2009). Enhancing students' understanding in calculus through writing. *International Electronic Journal of Mathematics Education, 4*, 36−55.

Kostos, K., & Shin, E. (2010). Using math journals to enhance second grades' communication of mathematics thinking. *Early Childhood Education Journal, 38*(3), 223-231. doi:10.1007/s10643-010-0390-4

Lee, E., & Hannafin, M. (2016). A design framework for enhancing engagement in student-centered learning: Own it, learn it, and share it. *Education Technology Research and Development, 64*(4), 707-734. doi:10.1007/s11433-015-9422-5

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grows (Ed.), *Handbook of Research on Mathematics Teaching and Learning* (pp. 575-596). New York: Macmillan.

Moran, A. S., Swanson, H. L., Gerber, M. M., & Fung, W. (2014). The effects of paraphrasing interventions on problem-solving accuracy for children at risk for math disabilities. *Learning Disabilities Research and Practice, 29*, 97−105. doi:10.1111/ldrp.12035

Moyer-Packenham, P. S., Baker, J., Westenskow, A., Anderson-Pence, K. L., Shumway, J. F., & Jordan, K. E. (2014). Predictors of achievement when virtual manipulatives are used for mathematics Instruction. *REDIMAT - Journal of Research In Mathematics Education*, *3*(2), 121-150.

National Council of Teachers of Mathematics. (2015). *Strategic use of technology in teaching and learning mathematics: A position of the National Council of Teachers of Mathematics*. Reston, VA: NCTM.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM.

Noonan, J. (1990). Readability problems presented by mathematics text. Early Child Development and Care, 54, 57-81. doi: 10.1080/0300443900540104

Organization for Economic Cooperation and Development [OECD] (2016, March). Is memorization a good strategy for learning mathematics? *PISA in Focus (Report No. 61)*. Paris, France: OECD.

Porras, D. (1994). Do your students digest mathematics like ice cream or like steak? Mathematics and Computer Education, 28(1), 6-15.

Pourdavood, R. G. & Liu, X. (2017). Pre-service elementary teachers' experiences, expectations, beliefs, and attitudes toward mathematics teaching and learning. *International Journal of Learning, Teaching and Educational Research, 16*(11), 1-27. doi:10.26803/ijlter.16.11.1

Pourdavood, R. G., & Wachira, P. (2015). Importance of mathematical communication and discourse in secondary classrooms. *Global Journal of Science Frontier Research, 15*(10), 9-20.

Pourdavood, R. G., Wachira, P., & Pitre, S. (2015). A case study of a cecondary mathematics teacher's beliefs and practices relative to NCTM principles : Implication for teacher education, curriculum change, and school reform. *Global Journal of Mathematics, 6*(2), 592-600.

Powell, S. R., Hebert, M. A., Cohen, J. A., Casa, T. M., & Firmender, J. M. (2017). A synthesis of mathematics writing: Assessments, interventions, and surveys. *Journal of Writing Research, 8*(3), 493-526. doi: 10.17239/jowr-2017.08.03.04

Rakes, C. R., Ronau, R. N., Niess, M. L., Driskell, S., Pugalee, D., & Bush, S. (2011, January). *Research in mathematics instructional technology: Current trends and future demands*. Symposium presented at the annual meeting of the Association of Mathematics Teacher Educators, Irvine, CA.

Rau, M. A., Aleven, V., & Rummel, N. (2015). Successful learning with multiple graphical representations and self-explanation prompts. *Journal of Educational Psychology, 107*(1), 30-46. doi:10.1037/a0037211

Rittle-Johnson, B., & Koedinger, K. R. (2009). Iterating between lessons on concepts and procedures can improve mathematics knowledge. *British Journal of Educational Psychology, 79*(3), 483-500. doi:10.1348 /000709908x398106

Rittle-Johnson, B., Siegler, R. S., & Alibali, M. W. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *Journal of Educational Psychology, 93*(2), 346-362. doi:10.1037 //0022-0663.93.2.346

Satsangi, R., Bouck, E. C., Taber-Doughty, T., Bofferding, L., & Roberts, C. A. (2016). Comparing the effectiveness of virtual and concrete manipulatives to teach algebra to secondary students with learning disabilities. *Learning Disability Quarterly*, *39*(4), 240-253. doi:10.1177/0731948716649754

Schneider, M. Rittle-Johnson, B., & Star, J. (2011). Relations between conceptual knowledge, procedural knowledge, and procedural flexibility in two samples differing in prior knowledge. *Developmental Psychology, 47*, 1525-1538. doi:10.1037/a0024997

Shuard, H., & Rothery, A. (1984). Children reading mathematics. London: John Murray.

Siegel, M., Borasi, R., Fonzi, J. M., Sanridge, L. G., & Smith, C. (1996). Using reading to construct mathematical meaning. In Elliot, P. C. & Kenney, M. J. (Eds.), Communication in mathematics: K – 12 and beyond (pp. 66-75). Reston, VA: National Council of Teachers of Mathematics.

Stuart, C., & Thurlow, D. (2000). Making it their own: Preservice teachers' experiences, beliefs, and classroom practices. *Journal of Teacher Education*, *51*(2), 113. doi:10.1177/002248710005100205

Tobin, K. (1986). Effects of teacher wait time on discourse characteristics in mathematics and language arts classes. *American Educational Research Journal, 23*(2), 191-200.

Tsuruda, G. (1994). *Putting it together: Middle school.* Portsmouth, NH: Heinemann.

Vasquez, S. & McCabe, T. W. (2002). The effect of calculator usage in the learning of basic skills. *Research and Teaching in Developmental Education 19*(1), 33-40.

Wachira, P., Pourdavood, R. G., Skitzki, R. (2013). Mathematics teacher's role in promoting classroom discourse. *International Journal for Mathematics Teaching and Learning.* Available from http://www.cimt.org.uk/ijmtl

Wheatley, K. F. (2012). How healthy motivation can help transform education. Encounter, 25(1), 41-47.

Wilcox, B., & Monreo, E. E. (2011). Integrating writing and mathematics. *The Reading Teacher, 64*(7), 521-529. doi:10. 1598/rt.64.7.6

Young, M. F., Slota, S., Cutter, A. B., Jalette, G., Mullin, G., Lai, B., & Yukhymenko, M. (2012). Our princess is in another castle: A review of trends in serious gaming. *Review of Educational Research, 82*(1), 61–89. doi: 10.3102/0034654312436980

Appendix 1
Pre-Service Teacher Survey

Age: _____ **Gender**: Male ______Female______

For items 1 through 45, identify your mathematical experiences when you were a K-12 student and your mathematical expectations in an adult workplace by rating each item on a scale from 1 (never) to 5 (always).

1	2	3	4	5
never	rarely	sometimes	often	always

Memorization
1. In elementary school, I memorized facts, rules or algorithms. ①②③④⑤
2. In middle school, I memorized facts, rules or algorithms. ①②③④⑤
3. In high school, I memorized facts, rules or algorithms. ①②③④⑤
4. In college, I memorized facts, rules or algorithms. ①②③④⑤
5. In an adult workplace, I would expect to memorize facts, rules or algorithms. ①②③④⑤

Technology
6. In elementary school, I used a calculator. ①②③④⑤
7. In middle school, I used a calculator. ①②③④⑤
8. In high school, I used a calculator. ①②③④⑤
9. In college, I used a calculator. ①②③④⑤
10. In an adult workplace, I would expect to use a calculator. ①②③④⑤

Measurement
11. In elementary school, I used rulers, scales or other measurement tools. ①②③④⑤
12. In middle school, I used rulers, scales or other measurement tools. ①②③④⑤
13. In high school, I used rulers, scales or other measurement tools. ①②③④⑤
14. In college, I used rulers, scales or other measurement tools. ①②③④⑤
15. In an adult workplace, I used rulers, scales or other measurement tools. ①②③④⑤

Manipulatives
16. In elementary school, I used tiles, blocks or other counters. ①②③④⑤
17. In middle school, I used tiles, blocks or other counters. ①②③④⑤
18. In high school, I used tiles, blocks or other counters. ①②③④⑤
19. In college, I used tiles, blocks or other counters. ①②③④⑤
20. In an adult workplace, I would expect to use tiles, blocks or other counters. ①②③④⑤

Multiple Representations
21. In elementary school, I made tables, charts, or graphs. ①②③④⑤
22. In middle school, I made tables, charts, or graphs. ①②③④⑤
23. In high school, I made tables, charts, or graphs. ①②③④⑤
24. In college, I made tables, charts, or graphs. ①②③④⑤
25. In an adult workplace, I would expect to make tables, charts, or graphs. ①②③④⑤

Reading
26. In elementary school, I read about mathematical ideas. ①②③④⑤
27. In middle school, I read about mathematical ideas. ①②③④⑤
28. In high school, I read about mathematical ideas. ①②③④⑤
29. In college, I read about mathematical ideas. ①②③④⑤
30. In an adult workplace, I would expect to read about mathematical ideas. ①②③④⑤

Discourse

31. In elementary school, I talked about mathematical ideas. ①②③④⑤
32. In middle school, I talked about mathematical ideas. ①②③④⑤
33. In high school, I talked about mathematical ideas. ①②③④⑤
34. In college, I talked about mathematical ideas. ①②③④⑤
35. In an adult workplace, I would expect to talk about mathematical ideas. ①②③④⑤

Illustration

36. In elementary school, I drew pictures representing mathematical ideas. ①②③④⑤
37. In middle school, I drew pictures representing mathematical ideas. ①②③④⑤
38. In high school, I drew pictures representing mathematical ideas. ①②③④⑤
39. In college, I drew pictures representing mathematical ideas. ①②③④⑤
40. In an adult workplace, I would expect to draw pictures representing mathematical ideas. ①②③④⑤

Communication

41. In elementary school, I wrote about mathematical ideas. ①②③④⑤
42. In middle school, I wrote about mathematical ideas. ①②③④⑤
43. In high school, I wrote about mathematical ideas. ①②③④⑤
44. In college, I wrote about mathematical ideas. ①②③④⑤
45. In an adult workplace, I would expect to write about mathematical ideas. ①②③④⑤

For items 46 through 48, please indicate the extent to which you agree or disagree with each statement on regarding your attitudes towards learning mathematics. There are no right or wrong answers. The scale ranges from 1 (*strongly disagree*) to 5 (*strongly agree*). Please circle the number that best describes what is true for you.

1	2	3	4	5
strongly disagree	disagree	neutral	agree	strongly agree

46. My interest level regarding mathematics is high.

1 2 3 4 5

47. My motivation level regarding mathematics is high.

1 2 3 4 5

48. My confidence level regarding mathematics is high.

1 2 3 4 5

For items 49 through 54, please indicate the extent to which you agree or disagree with each statement on regarding your beliefs about teaching mathematics. There are no right or wrong answers. The scale ranges from 1 (*strongly disagree*) to 5 (*strongly agree*). Please circle the number that best describes what is true for you.

1	2	3	4	5
strongly disagree	disagree	neutral	agree	strongly agree

49. Students should view their teacher as a facilitator of learning rather than the dispenser of knowledge.

1	2	3	4	5

50. Students should be allowed to invent ways to solve problems before the teacher demonstrates how to solve the problems.

1	2	3	4	5

51.Content should be presented to students in such a way that they can discover relationships for themselves.

1	2	3	4	5

52. Allowing students to discuss their thinking helps them to make sense of the content.

1	2	3	4	5

53. The instructional sequence of topics should be determined by the order in which students naturally acquire concepts.

1	2	3	4	5

54. When selecting the next topic to be taught, a significant consideration is what students already know.

1	2	3	4	5