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Teaching the Law of Cosines in Advanced 10th Grade Geometry Textbook by Way of Problem-Based Learning in Vietnam

Ngoc-Giang Nguyen
Doctor of Banking University of Ho Chi Minh City
36 Ton That Dam, Nguyen Thai Binh ward, district 1,
Ho Chi Minh City, Vietnam

Abstract. The law of Cosines is an essential theorem in mathematics in particular and in science in general. The Cosine Rule allows us to calculate the length of one side of a triangle when we know two sides and the angle between the other two sides. This theorem is one of the first theorems when people want to build trigonometric systems. Because of such importance, many people studied the Law of Cosines. In Vietnam, they taught the Law of Cosines in the 10th-grade Geometry program. It appears in Basic and Advanced sections, after having learned some concepts about vector operations. Although it is a famous theorem, there are not many documents mentioning how to teach the Law of Cosines effectively. We need to understand the relationship between mathematics and other fields expressed by the Law of Cosines, as well as to apply the method of problem-based learning in teaching this theorem.

Keywords: Law of Cosines; problem-based learning; a real problem; Informatics; Physics.

1. Introduction

The rapid development of science and technology requires a lot of innovation in Education and Training. In Vietnam today, the way of teaching is much more different than before. In the past, instruction focused on catching and understanding the knowledge of students while teaching nowadays focuses on the development of students' competency. Many new teaching methods are applied, including the teaching method of discovering and solving problems. The process of problem-based learning has the advantages of promoting learners' activeness and creativity, creating an active learning environment, encouraging learners to explore. Learners through activities of problem-based learning have formed knowledge for themselves. Therefore, education is bolded in thinking of learners. Learners will remember longer and know how to apply knowledge into real-life flexibly.

Many people have studied problem-based learning. For example, how did Strobel & van Barbneveld (2009) study the effects of problem detection and problem-solving on students? Ferreira and Trudel (2012) explored the impact of problem-finding and problem-solving methods on student attitudes in science, problem-solving skills, and community in the class. Hmelo - Silver (2004) studied what students learn, how students learn in problem-based learning, while Bligh (1995) referred to small group teaching in problem-discovering and problem-solving teaching methods. Besides, Beringer (2007) noted to the application of the way of problem-based learning through research investigation while Sahar, Sani and Malau (2017) reviewed the impact of problem-based learning on students' ability to solve math problems at Senior High School. Also, Do and Le (2017) studied the application of the teaching method of problem-based learning in teaching similar triangular subjects (8^{th}-grade math) in Middle Schools in Vietnam. Phan and Nguyen (2017) developed a scale and set of tools to assess students' problem-solving ability through project teaching, while Tran (2012) researched on problem-solving capacity and gained many remarkable results.

Currently, there has been much research on teaching the Law of Cosines. For example, Abramson et al. (2017) proposed a method of teaching the Law of Cosines by empirical statistics of specific numbers to draw this law. Doan, Van Pham and Bui (2019) came up with the application of the scalar product and derived the Law of Cosines, from which he proposed its applications.

Although there have been much research on teaching problem-based learning, there has been a few or no research on applying the teaching method of problem-based learning in teaching this law. Therefore, we give how to interpret the law of Cosines by problem-based learning.

2. Content

2.1. Teaching in the problem-based learning

Problem-based learning is a positive teaching perspective, promoting internal resources of learners. Problem-based learning has many different views. In teaching in problem-based learning, teacher gives to students a problematic situation, which is a situation that contains cognitive conflict, through problem-based learning, they acquire knowledge, skills and cognitive methods (Meier & Nguyen, 2011). Nguyen (2004) supposes that, in problem-based learning, teachers create problem-solving situations, guide students in problem detection, act voluntarily, positively and proactively and creatively to solve problems through which students build knowledge, practice skills, and achieve other learning goals.

Problem-based learning is a student-centred educational method that uses problem-solving as the starting point for learning (Bligh, 1995). The problem-based learning approach had a significant effect on the creative flexibility of advertising design students (Cheung, 2011). Doig (1993) reported that problem-based learning is designed to develop problem-solving strategies, disciplinary knowledge bases and skills simultaneously by placing students in the active role of problem-solvers. Problem-based learning is a dynamic learning method based on the use of ill-structured problems as a stimulus for learning (Barrows, 2000). Ulger (2018) reported that problem-based learning has a significant effect on creative thinking. Similarly, Chan (2013) stated that problem-based learning increased critical thinking and creativity. Nargundkar, Samaddar and Mukhopadhyay (2014) determined that Problem-based learning effectively enhances essential thinking of business school students. Chian, Bridges and Lo (2019) referred to the triple jump in Problem-Based Learning. Sullivan (2019) reported the challenge of Problem-based learning, while Purichia (2019) determined the inquiry approach to based learning. Besides, Jonassen (2011) referred to problem-solving in Problem-based learning.

From the above comments, we define problem-based learning as follows: *Problem-based learning is a process that teacher creates problematic situations. Teacher leads learners to overcome difficult obstacles in learning. He gradually clarifies the issues raised and helps students to learn positively and proactively so that students will not be dependent. Students create knowledge for themselves, create and develop understanding in a sustainably and creatively.*

2.2. The process of applying the teaching method of problem-based learning

According to Nguyen (2004), the core of the teaching method of problem-based learning is the control of teachers in the process of teaching to help students research and solve problems. The teaching process is carried out in 4 steps:

- Step 1: Detector or investigate the issue

From a problem-causing situation created by a teacher, students can relate to ways of thinking, exploring, anticipating, and finding problematic situations; Teachers explain and correct the problem; Students state the problem and set a goal to solve it.

- Step 2: Find a solution

Find a way to solve the problem. First, clarify the relationship between the known and the sought factor. Next, propose and implement solutions by collecting, organizing data, mobilizing knowledge, using methods, cognitive techniques, searching, deducing and thinking towards the goal, turning strange things into familiar, etc. We rejects the proposed direction if not reasonable, find a cheaper solution. Next is the solution formation. Finally, check the answer.

- Step 3: Present the solution

When the problem was solved, the student presents the entire process of solution implementation. During the presentation, it is necessary to comply with regulations such as: clearly stating assumptions, conclusions, logical and clear arguments. If the problem is a given problem, then there is no need to report the question again.

- Step 4: In-depth study of solutions

Find out the applicability of the result; Propose relevant new issues by analogy, generalization, problem reversing, etc., and problem-solving (if possible).

2.3. Developing a measure of the ability to detect and solve problems

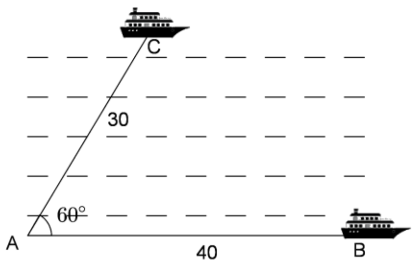
Phan and Nguyen (2017) provided the following steps of problem detection and resolution:

Table 1. The degree of expression

Component capacity	**Student expression**	**The degree of expression**			
		0	**1**	**2**	**3**
Issue detection	1. Students can raise issues that teacher gives to them in the assigned task.	Students cannot raise issues.	Students may raise incomplete issues.	Students can raise issues more thoroughly but slowly, thanks to the guidance of teachers.	Students can raise issues themselves thoroughly and quickly.
Proposing a solution	2. Students can provide relevant information.	Students cannot provide relevant information.	Students provide incomplete relevant information.	Students can state all relevant information.	Students can fully state relevant information accurately and scientifically.
	3. Students can propose a solution to the problem.	Students cannot propose a solution to the problem.	Students can propose solutions to problems, but those solutions are less feasible and ineffective.	Students can propose feasible solutions.	Students can propose creative solutions that can solve problems as quickly as possible.
Problem solving	4. Performing problem solving.	Students cannot solve problems so they cannot create any products.	Students who are confused when solving problems should create defective products in both form and content.	Students can solve problems well, create products with useful content but not in good form.	Students can solve problems and create outstanding work in both content and form.
Evaluating implementation results	5. Self-assessment of implementation results.	Students cannot self-assess.	Students have not stated precisely the advantages and limitations of the results.	Students can accurately state the advantages and limitations of performance, but they are unfounded and have not gained experience.	Students can accurately state the advantages and limitations of performance, have valid grounds and draw experience.

3. Illustrative example applying the teaching method of problem-based learning into teaching the Law of Cosines

- Step 1. Detecting or investigating the problem
Teacher: In fact, we encounter the following problem:
Example 1
The two ships, from the same position A, went straight in two directions and formed an angle of 60 degrees. Ship B ran at 20 knots. Ship C runs at 15 knots. After 2 hours, how many nautical miles are the two ships apart? (1 nautical mile = 1,852 km) (Doan et al., 2019)



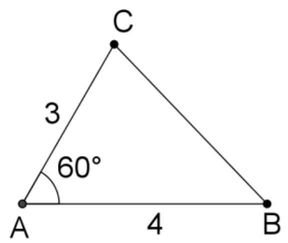
Teacher: Given that the velocity of train B is $v = 20$ knots/hour and the time is 2 hours, how much is the distance AB?
Student: $AB = v.t = 20.2 = 40$ (nautical miles).
Teacher: Given that the velocity of train A is $v = 15$ knots/hour and the time is 2 hours, how much is the distance AB?
Student: $AB = v.t = 15.2 = 30$ (knots).
Teacher: How is the problem now presented as a simple math problem?
Student: This is my statement:

Example 2
Given that the triangle ABC have sides $AB = 40; AC = 30; \widehat{BAC} = 60^{\circ}$. *Please calculate the edge of BC?*
Teacher: Please draw a triangle, measure the lengths of the edges and the angles, fill in the following table:

Table 2. Law of Cosines 1

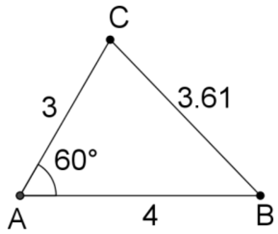
a	a^2	b	b^2	c	c^2	$\hat{A}$	$b^2 + c^2 - 2bc\cos A$

Teacher: For the first measurement, please use a computer, explicitly using GeoGebra dynamic geometry software, measure the length of the BC side when you know $AB = 4$, $AC = 3$ and the angle $\widehat{BAC} = 60^{\circ}$.

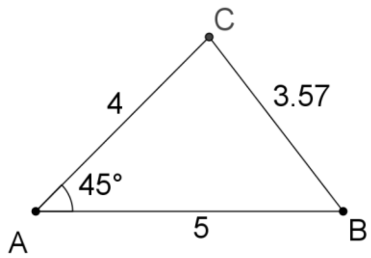


Students: We do not know how to draw and measure shapes in GeoGebra software.
Teacher: I will draw and measure BC on GeoGebra software as follows.

- Draw a line AB with a length of 4.
- Draw a segment of AC with a length of 3 so that $\hat{BAC} = 60^{o}$.
- Hide unnecessary objects.
- Measure the length of the segment BC, we have $BC = 3.61$.

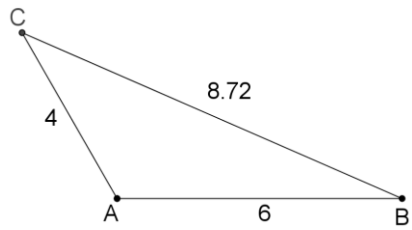


Teacher: For the second measurement, please use a computer, explicitly using GeoGebra dynamic geometry software, measure the length of the BC edge when you know $AB = 5$, $AC = 4$ and angle $\hat{BAC} = 45^{o}$.



Student: I have measured $BC = 3.57$.

Teacher: For the third measurement, please use a computer, explicitly using GeoGebra dynamic geometry software to measure the length of the BC side when you know that $AB = 6$, $AC = 4$ and the angle $\hat{BAC} = 120^{o}$.



Student: I have measured $BC = 8.72$.

Teacher: Using the given data for each measurement, please fill in the following table:

Table 3. Law of Cosines 2

a	a^2	b	b^2	c	c^2	$\hat{A}$	$b^2 + c^2 - 2bc\cos A$
3.61		3		4		60^{o}	
3.57		4		5		45^{o}	
8.72		4		6		120^{o}	

Student: I have measured the following:

Table 2. Law of Cosines 3

a	a^2	b	b^2	c	c^2	$\hat{A}$	$b^2 + c^2 - 2bc\cos A$
3.61	13	3	9	4	16	60^o	13
3.57	12.7	4	16	5	25	45^o	12.7
8.72	76	4	16	6	36	120^o	76

Teacher: What do you think about a^2 and $b^2 + c^2 - 2bc\cos A$?

Student: These two results are equal.

Teacher: Right! This thing is also the content of the Law of Cosines. Can you state the Law of Cosines?

Student:

In a triangle with $BC = a, CA = b, AB = c,$ we have

$a^2 = b^2 + c^2 - 2bc\cos A;$

$b^2 = c^2 + a^2 - 2ca\cos B;$

$c^2 = a^2 + b^2 - 2ab\cos C.$

- Step 2. Find a solution

Teacher: Which expression is equal to $bc\cos A$?

Students: Expression $\vec{b}\,.\,\vec{c}$.

Teacher: Please write the expression $b^2 + c^2 - 2bc\cos A$ in vector form.

Student: $\vec{b}^2 + \vec{c}^2 - 2\vec{b}.\vec{c} = (\vec{b} - \vec{c})^2.$

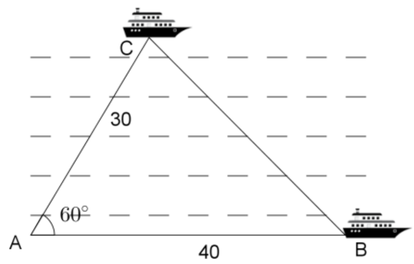
- Step 3: Present the solution

Teacher: Can you prove the Law of Cosines?

Student: We have:

$a^2 = \overrightarrow{BC}^2 = (\overrightarrow{AC} - \overrightarrow{AB})^2 = AC^2 + AB^2 - 2AC.AB\cos A = b^2 + c^2 - 2bc\cos A.$

Teacher: Can you use the Law of Cosines to prove Example 1?



After 2 hours, ship B went 40 nautical miles, and ship C went 30 nautical miles.

So the triangle ABC has $AB = 40,\ AC = 30,\ \hat{A} = 60^o.$

Applying the Law of Cosines into the triangle $ABC,$ we have:

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 30^2 + 40^2 - 2\,.\,30\,.\,40\,.\,\cos 60^o$$
$$= 900 + 1600 - 1200 = 1300.$$

So $BC = \sqrt{1300} \approx 36$ (nautical miles).

After 2 hours, the two ships are about 36 nautical miles apart.

Teacher: If $\hat{A} = 90^o$ then we obtain the Pythagorean theorem. Please state this result.

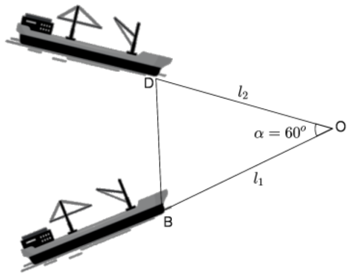
Student: In a right triangle with right angle A with $BC = a, CA = b, AB = c,$ we have
$a^2 = b^2 + c^2.$

- Step 4: Find out the solution more deeply

Teacher: The Cosine Rule has many other applications in physics. Can you solve the following problem using the Law of Cosines?

Example 3

Two ships were moving at the same velocity v heading to O along the paths -the straight lines that join at an angle of $\alpha = 60^o$. Determine the minimum distance between ships. Initially, they were about $l_1 = 20km$ and $l_2 = 30km$ from O. (Le, 2018).



Student: I will solve the problem like this (after a while of thinking):

Suppose t is the travel time of two ships at the minimum distance; d is the distance between the two ships. The distance between the two ships is:

$d = BD = \sqrt{OB^2 + OD^2 - 2OB.OD.\cos 60^o}.$

So

$$d^2 = OB^2 + OD^2 - 2OB.OD.\cos 60^o$$

$$\Rightarrow d^2 = (l_1 - vt)^2 + (l_2 - vt)^2 - 2(l_1 - vt).(l_2 - vt).\frac{1}{2}$$

$$\Rightarrow d^2 = l_1^2 - 2l_1vt + v^2t^2 + l_2^2 - 2l_2vt + v^2t^2 - l_1l_2 + l_1vt + l_2vt - v^2t^2$$

$$\Rightarrow d^2 = l_1^2 + l_2^2 - l_1l_2 + \left(\frac{l_1 + l_2}{2} - vt\right)^2 - \left(\frac{l_1 + l_2}{2}\right)^2$$

$$\Rightarrow d^2 = \frac{3}{4}\left(l_1^2 + l_2^2\right) - \frac{3}{2}l_1l_2 + \left(\frac{l_1 + l_2}{2} - vt\right)^2.$$

If $d = d_{\min}$ then $\left(\frac{l_1 + l_2}{2} - vt\right)^2 = 0$ and then

$$d = d_{\min} = \sqrt{\frac{3}{4}\left(l_1^2 + l_2^2\right) - \frac{3}{2}l_1l_2}$$

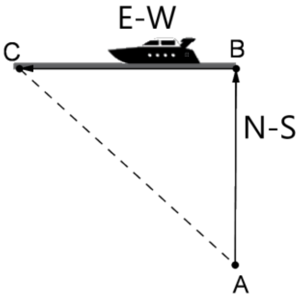
$$\Rightarrow d = d_{\min} = \sqrt{\frac{3}{4}.(20^2 + 30^2) - \frac{3}{2}.20.30} = \sqrt{75} \approx 8,66\ (km).$$

So the minimum interval between ships is $d_{\min} = 8,66\ (km).$

Teacher: The particular case of the Law of Cosines is the Pythagorean theorem. Please solve the following problem:

Example 4

A canoe left the station moving steadily. At first, the ship ran in the south-north direction for 2 minutes and 40 seconds, then immediately turned to the east-west direction and ran for another 2 minutes at the same speed as before and stopped. The distance from the starting place to the stop is 1 km. Calculate the speed of that canoe.



Nguyen, P. D, 2018)

Student: I will solve the problem like this:

We have: 1 minute 40 seconds = 160 s; 2 minutes = 120 s; 1 km = 1000 m.

Assume A is the starting point, B is the starting point, and C is the stop. We have:

$$AC^2 = AB^2 + BC^2 \Rightarrow AC^2 = (vt_1)^2 + (vt_2)^2.$$

We infer: $v = \dfrac{AC}{\sqrt{t_1^2 + t_2^2}} = \dfrac{1000}{\sqrt{160^2 + 120^2}} = 5\ (m/s) = 18\ (km/h).$

So the speed of the canoe is $v = 18\ km/h.$

Teacher: The Law of Cosines also has many applications in informatics. The following example is an illustration of drawing a figure using the Law of Cosines on GeoGebra software to help find the exact shape of the locus.

Example 5

Given the angle $\widehat{xOy} = \alpha$ (α given). The isosceles triangle ABC has $AB = AC = a$ (constant), $\hat{A} = 180^o - \alpha$; B and C move on Ox and Oy rays, respectively (O and A are on two halves of the plane opposite each other with the edge BC). Find the locus of point A. (Nguyen, 2010)

Teacher: We draw the shape and find the locus of point A on GeoGebra software:

. Step 1. Drawing the shape

. Draw the corner $\widehat{xOy}$.

. Get point B on the Ox ray.

. Construct a segment PQ with length a.

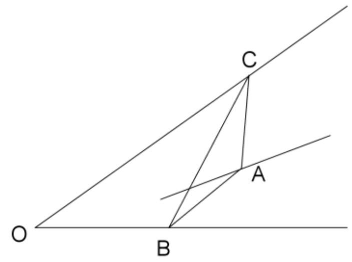
. Construct a segment MN whose length is $a\sqrt{2*(1+\cos\widehat{xOy})}$.

. The circle with center B and radius equal to MN cuts Oy at C.

. The circle $(C; PQ)$ intersects $(B; PQ)$ at A, such that O and A lie on two halves of the plane opposite each other with the edge BC.

- Step 2. Locus

. Trace point A, move point B, we obtain a set of marks A, which is a straight line.



(Nguyen, N. G, 2010)
Teacher: Here, we use the Law of Cosines:
$BC^2 = AB^2 + AC^2 - 2AB.AC\cos A = 2a^2 + 2a^2\cos a$.

We infer $BC = a\sqrt{2(1 + \cos \widehat{xOy})}$. Hence we have the above construction.

Teacher: The above example is an application of the Law of Cosines in informatics. You go home to find more other uses of the Law of Cosines.

4. Educational experiment

4.1. Purpose of the pedagogical experiment

The empirical goal is to test the feasibility and effectiveness of teaching the Law of Cosines according to problem-based learning.

4.2. Experimental content

We carry out the experimental teaching on the Law of Cosines in Advanced 10th-Grade Geometry program. Specifically, we conducted experiments as follows:
- Teaching how to use essential GeoGebra software and the Law of Cosines (2 periods).

During the experiment, we had students do two tests to evaluate and compare qualitative and quantitative aspects, etc.
The experimental teacher is also a control teacher and helped by her teacher, Do.

4.3. Experimental organization

4.3.1. Experimental object

The experiment conducted at Truong Chinh High School, Ho Chi Minh City.
Experimental class: 10A05 has 32 students.
Control class: 10A06 has 30 students.
Generally, in both classes, the conditions are relatively equal (number of males, females, ages, learning conditions, etc.).
The results of the quality survey at the beginning of the school year showed that the math quality of the two classes is relatively uniform.

4.3.2. Experimental process

For the experimental class, we prepare lessons with educational intent using GeoGebra software to improve the effectiveness of the teaching process.
For the control class, the teacher conducts teaching regularly. Experimental teaching and controlled teaching perform in parallel with the school's teaching schedule. We prepare the tests and speak to the control teacher before letting the students take the test.

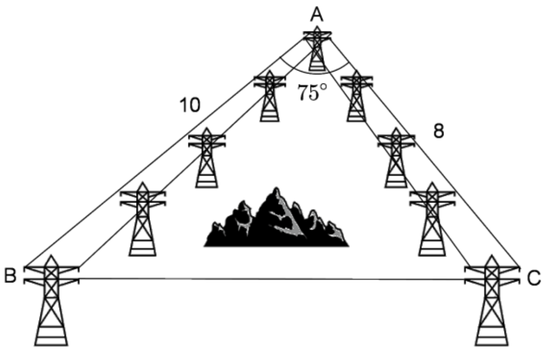
Here we list some of the exercises taught in some lessons to correct practices for students in experimental class.

4.3.3. Test results

During the experiment, we conducted two tests.

Test number 1 (exam time 10 minutes)

A high-voltage line connecting straight from position A to position B is 10 km long. The distance from position A to position C is 8 km, the angle created by the above two lines is equal to 75^o. Calculate the distance from position B to position C (as shown in the figure). (Doan et al., 2019)

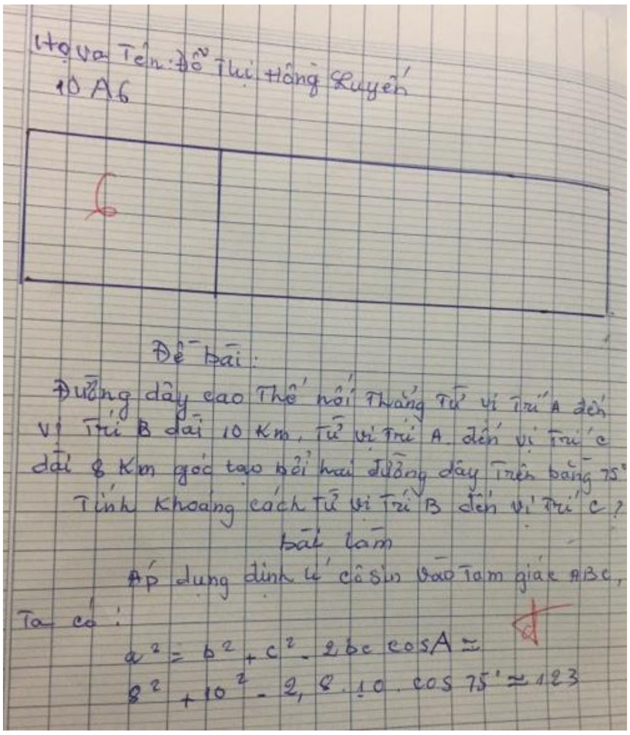


The answer and the scale of test 1 are as follows:

Table 4. Scale score

Content	Score
Applying the Law of Cosine to the triangle ABC, we have: $a^2 = b^2 + c^2 - 2bc\cos A \approx 8^2 + 10^2 - 2\,.\,8\,.\,10\,.\,\cos 75^o \approx 123.$	Six marks
We infer: $a \approx 11\ (km)$. Therefore, the distance from B to C is approximately $11\ km$.	Four marks

A student does the test as follows (Written in Vietnamese):


Họ và Tên: Đỗ Thị Hồng Luyến
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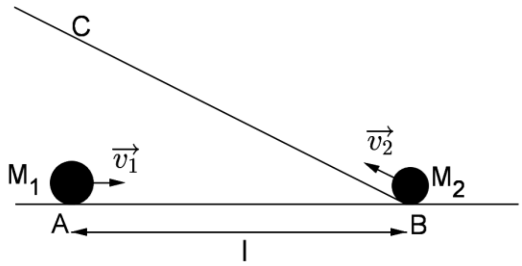
Đề bài:
Đường dây cao thế nối thẳng từ vị trí A đến vị trí B dài 10 Km, từ vị trí A đến vị trí C dài 8 Km góc tạo bởi hai đường dây trên bằng 75°
Tính khoảng cách từ vị trí B đến vị trí C?
Bài làm
Áp dụng định lý cô sin vào tam giác ABC,
Ta có:
$a^2 = b^2 + c^2 - 2bc \cos A \simeq$
$8^2 + 10^2 - 2 . 8 . 10 . \cos 75° \simeq 123$

Test 2 (test time 20 minutes)

There are two mass points M_1 *and* M_2 *which move in a straight line with velocity* v_1 *and* v_2*, respectively. In the beginning, the mass point* M_1 *at A is going to B, the mass point* M_2 *at B is going to C, and they are separated by a distance* l *as shown in the figure.*

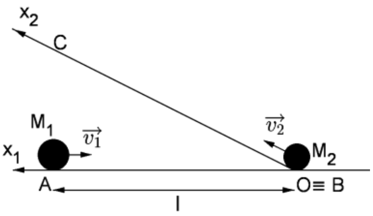


a) Calculate the shortest distance between two mass points and the time to achieve this distance.

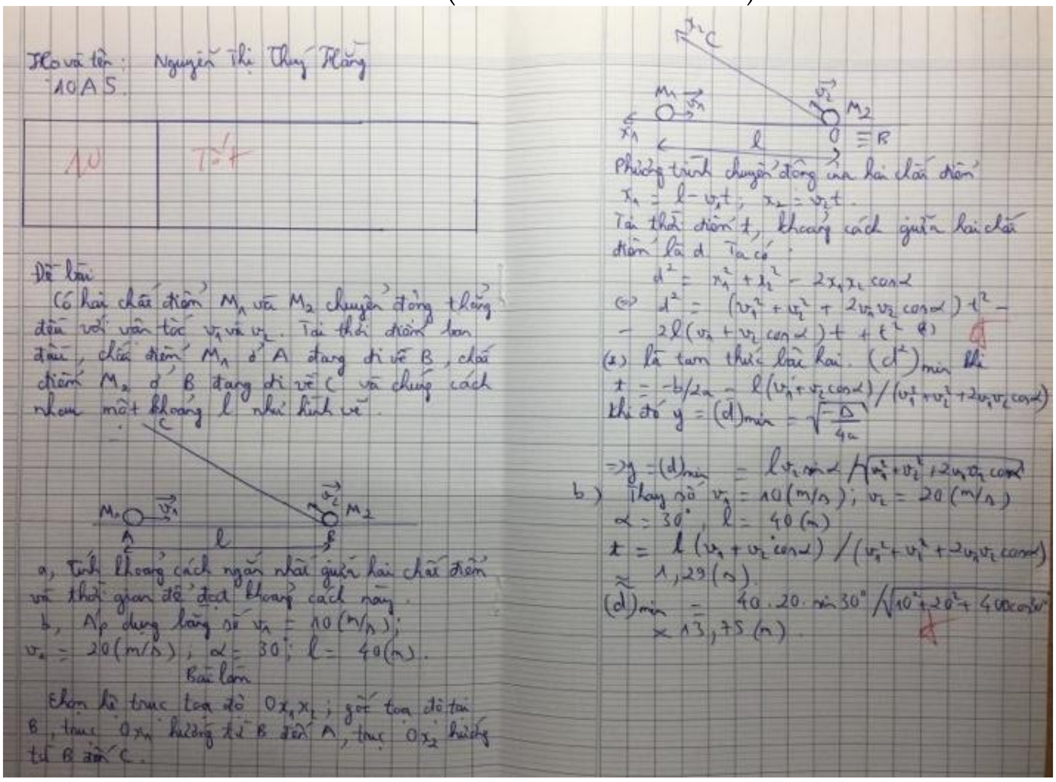
b) Apply with the number $v_1 = 10\ (m/s);\ v_2 = 20\ (m/s);\ \alpha = 30^o;\ l = 40\ (m)$. (Le, 2018)

The answer and the scale of test 2 are as follows:

Table 5. Scale score

Content	Score
Select the coordinate system Ox_1x_2; origin at B, Ox_1 axis oriented from B to A; Ox_2 axis oriented from B to C.  The motion equation of the two material mass points: $x_1 = l - v_1 t;\ x_2 = v_2 t.$	Two marks
At the time t, the distance between the two material mass points is d. We have: $d^2 = x_1^2 + x_2^2 - 2x_1x_2\cos\alpha$ $\Leftrightarrow d^2 = (l - v_1t)^2 + (v_2t)^2 - 2(l - v_1t)(v_2t)\cos\alpha$ $\Leftrightarrow d^2 = \left(v_1^2 + v_2^2 + 2v_1v_2\cos\alpha\right)t^2 - 2l\left(v_1 + v_2\cos\alpha\right)t + l^2.$	Two marks
Set $\begin{cases} a = v_1^2 + v_2^2 + 2v_1v_2\cos\alpha \\ b = -2l(v_1 + v_2\cos\alpha) \\ \mathrm{c} = l^2 \end{cases} \Rightarrow d^2 = at^2 + bt + l^2$ (*). (*) is a quadratic triple, so $y = (d^2)_{\min}$ when $t = \dfrac{-b}{2a} = \dfrac{l(v_1 + v_2\cos\alpha)}{(v_1^2 + v_2^2 + 2v_1v_2\cos\alpha)}.$ Then: $y = (d)_{\min} = \sqrt{\dfrac{-\Delta}{4a}}.$ $y = (d)_{\min} = \sqrt{-\dfrac{\left(2l(v_1 + v_2\cos\alpha)\right)^2 - 4l^2(v_1^2 + v_2^2 + 2v_1v_2\cos\alpha)}{4\left(v_1^2 + v_2^2 + 2v_1v_2\cos\alpha\right)}}$ $\Rightarrow y = (d)_{\min} = \dfrac{lv_2\sin\alpha}{\sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos\alpha}}.$	Three marks
b) Replacing the numbers: $v_1 = 10\ (m/s);\ v_2 = 20\ (m/s);\ \alpha = 30^o,\ l = 40\ (m).$ $t = \dfrac{l(v_1 + v_2\cos\alpha)}{(v_1^2 + v_2^2 + 2v_1v_2\cos\alpha)} = \dfrac{40.(10 + 20\cos 30^o)}{(10^2 + 20^2 + 2.10.20.\cos 30^o)} \approx 1{,}29\ (s).$ $(d)_{\min} = \dfrac{lv_2\sin\alpha}{\sqrt{v_1^2 + v_2^2 + 2v_1v_2\cos\alpha}} =$ $\dfrac{40.20.\sin 30^o}{\sqrt{10^2 + 20^2 + 2.10.20.\cos 30^o}} \approx 13{,}75\ (m).$	Three marks

A student does the test as follows (Written in Vietnamese):



4.4. Evaluating experimental results

4.4.1. Qualitative evaluation

Before experimenting:

- The competency of associating with the use of theorems and properties is still limited. When encountering practical problems, many students do not know where to start (ask homeroom teacher).
- Most students have difficulty applying the knowledge of the Law of Cosines in physics problems.

After experimenting:

- Students master the way to solve practical problems using the Law of Cosine.
- Students are interested and confident in solving physics problems by using the Law of Cosines.
- Students are interested in finding more examples of applying the Law of Cosines.

4.4.2. Quantitative evaluation

The two following tables shown the test results of experimental and control classes:

Table 6. Test result number 1

Classes \ Score	3	4	5	6	7	8	9	10	The sum of scores
Experimental	1	3	8	7	5	5	2	1	32
Control	0	4	8	9	4	4	1	0	30

The experimental class has 87.5% of the average score or higher, of which 40,625% is quite good (score of 7 or higher) with one student getting a perfect score, two students scoring 9.

The control class has 60% of the average score or higher, of which 30% is quite good (score of 7 or above), no student is achieving a perfect score, one student scoring 9.

Table 7. Test result number 2

Score / Classes	3	4	5	6	7	8	9	10	The sum of scores
Experimental	1	3	7	6	4	6	3	2	32
Control	1	3	8	8	4	4	1	1	30

The experimental class has 87.5% of the average score or higher, with 46,875% of a reasonably good score (score of 7 or higher) with two students achieving perfect scores, three students achieving nine marks.

The control class has 90% of the average score or higher, of which 33.33% is quite good (7 marks or more), one student has a perfect score, one student has a score of 9.

4.4.3. Conclusions about the experiment

Both results of tests show that the results of the experimental classes are higher than those of the control classes, especially the number of tests with good grades is a lot higher. The reason is that the experimental class students often practice how to learn by problem-based learning and solving practical problems.

The results initially obtained through the educational experiment allow to conclude that: The results initially obtained through the pedagogical experiment allow to conclude that: If we teach the Law of Cosines by the problem-based learning then it makes students feel interesting. It attracts students to mathematical activities voluntarily and positively. It stimulates curiosity and a passion for self-research. This method helps students to understand the basics knowledge better. From it makes of students to have the habits of independent thinking, solve problematic situations and clarify problems. These thing show the effectiveness of applying the method of problem-based learning in high schools.

Therefore the purpose of the educational experiment has been achieved, and the scientific hypothesis mentioned has proved experimentally.

5. Common conclusion

Problem-based learning is a positive teaching method, promoting the inner motivation of learners. Learners learn knowledge through problem-solving questions. Teachers present educational situations and lead students from pure understanding to sophisticated understanding. By using visual media such as GeoGebra's dynamic geometry software, the teachers make the lesson exciting. Students are excited to discover new experience with the help of this dynamic geometry software. Students fell that they gain knowledge by themselves and create understanding for themselves. Therefore, students will remember knowledge longer, smoothly and flexibly in applying their awareness into practice. The Cosine Rule is essential universal knowledge. This theorem is the basic knowledge of trigonometry. However, in Vietnam, the way of teaching this law is mainly the traditional way of teaching. The teacher presents the law, then illustrates to the applicable problems. This teaching method is dynamic, not encouraging independent and creative thinking of learners. By submitting a way of teaching about the Law of Cosines by the teaching method to detect and solve problems, we realize that the educational effect is much higher. These things prove that the problem-based teaching is better than the traditional teaching method for the Law of Cosines.

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