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Teachers' Beliefs about the Nature, Teaching and Learning of Mathematics and Sources of these Beliefs

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Abstract. This study aimed at investigating the beliefs of mathematics teachers and the relationship of these beliefs with their teaching practices, and the acquisition sources of these beliefs. The study population comprised (100) male and female teachers who were selected randomly out of those who teach the 12th graders. The study applied quantitative and qualitative methods together. The data were collected through the beliefs questionnaire, note card and interview card. The results showed that the teachers had a blend of swaying beliefs between constructivism and behaviourism about the nature of mathematics and math teaching. There was a lack of compatibility between the teachers' beliefs and their teaching practices. The results further showed that the primary sources of mathematics teachers' acquisition of their beliefs were: the social surrounding, former teachers, university studies, technology, media, self-reflection, personal experience, professional colleagueship, scientist, religious orientations, educational supervisors, examinations system and school principals.

Keywords: Teachers' beliefs; Nature of mathematics; School mathematics; Belief systems; Teaching practices.

1. Introduction

Preparing the teachers for the twenty-first century is not limited to developing their knowledge and skills required for their industrial assignments. Instead, the integrative understanding between teacher preparation and training requires, at first, calls for focusing on the development of his views about the entire components of the educational process. At that point, the study of the teachers' beliefs emerges as one of the underlying mechanisms to achieve these objectives. Studies of (Beswick, 2012; Charalambous & Philippou, 2019; Cross, 2015; Jong & Hodges, 2013; Minarni et al, 2018) indicated that the teaching practices of mathematics teachers are related to his beliefs. Cross (2015) defined the beliefs, generally, as many thoughts and conscious and unconscious perceptions about

the world; the individual's self and his place in this world. He indicated that beliefs develop through the individual's membership in the social groups.

Manderfeld and Siller(2019) define it as a pool of perceptions that the individuals hold, which are useful, impacting and acquired from the daily experience; or internal inherent in the personality of the individual personality. Rott (2019) adds that the beliefs influence the individual's abilities and direct his decisions about what he believes about the world and the surrounding things. They are more resistant to the change than the feelings, tendencies and trends.

Schoen and LaVenia (2019) indicate that the belief system is developed with math teachers before he begins his career life. He holds many beliefs about math nature and its teaching-learning. He is further influenced by his school and university experiences in learning mathematics. The belief system develops, and even maybe changed when the teacher practices math teaching, influenced by his personal experience and his career colleagues experiences as well. His students play an essential role in formulating and modifying his beliefs. The beliefs of mathematics teachers vary due to these influences. Some believe that mathematics is a mere knowledge that could be presented smoothly and easily. Others believe that mathematics is not efficiently understandable by the students without direction and intervention of the teacher.

Purnomo, Suryadi and Darwish (2017) pointed out to two kinds of beliefs about mathematics, learning and teaching thereof. The first is the traditional beliefs that emphasize the memory and training to achieve the mastery level. The second is the fundamental beliefs that focus on understanding and perception. The teacher whose beliefs are focused on memory believes that math learning will not be achieved unless he teaches math concepts. Thus, such a teaching is superficial due to the teachers' focus on training and keeping steps to reach the skill, unaware of the students' mistakes or their thinking manner. Meanwhile, the teacher who believes in the importance of the students' understanding of math experience and information perceives that it is not enough to be confined by the cognitive level. Instead, the previous knowledge must be linked to the new by providing opportunities for the students to practice math on their own. The teacher does not only focus on the conceptual and procedural understanding, but also overruns this to make the students practice math experience and apply them in their lives.

Beswick (2012) sees that math teacher's feelings towards mathematics nature determine his view and beliefs about mathematics teaching, the teaching style he prefers, the role of the students in math learning, and the quality of the appropriate math activities. (Jung, Zhang & Chiang, 2019) indicated to these points, stating that the beliefs of the teachers about mathematics influence the manner they present the content to the students. Therefore, the ways the content is presented point out to the teachers' beliefs about mathematics.

In a related context (Cobanoglu, Capa-Aydin & Yildirim, 2019) stated that the delicate sources of the beliefs and their acquisition mechanisms with the teachers

are still unclear. They maybe acquired through the teacher's bringing-up, the surrounding environment and the nature of the community. And, perhaps through contemplation in the experience he owns, and the experiments and practices he performs during his teaching. Still, the social bringing-up at schools may be one of the acquisition sources. Beliefs are the reflection of life experiences and may arise from personal experience or experience in teaching, as well as from the teacher's pedagogy.

Results of Memnun and Hart (2014) revealed that the beliefs of mathematics teachers in the United States of America are more consistent with the constructivism than those of their peers in Turkey. Study of Dede and Uysal (2012) showed that the beliefs of the teachers about mathematics agree with this modern view, with the focus being around the learner. In the same context, a study of (Zakaria & Maat, 2012) showed that the teachers hold non-traditional beliefs. They think that the role of the student is to discover the mathematical knowledge, and the role of the teacher is to direct and encourage the students in explaining their thinking. Reviews of (Bewick, 2005; Boz, 2008) showed that most teachers hold beliefs consistent with constructive outlook. Studies of (Shavarani & Savisi, 2007; Zacharos, Koliopoulos, Dokimaki & Kassoumi, 2007) showed that mathematics teachers are in line with the traditional outlook. Study (Gol & Duru, 2016) showed contradictions in the teachers' beliefs about mathematics, on the one hand, and about teaching it, on the other.

Results of (Gales & Yan, 2001; Roehrig, Turner, Grove, Schneider & Liu, 2009) emphasized the existence of compatibility between the beliefs and the teaching practices. Results of the studies of (Hilal & Ucar, 2010; Sapkova, 2014) pointed to the existence of contradictions between the teachers' beliefs and the teaching practices. Finally, Studies of (Beswick, 2012; Zakaria & Maat, 2012) showed a correlation relationship between the beliefs and the teaching practices.

Despite this interest and great efforts in researching and interpreting the beliefs of teachers, and trying to develop them to suit the modern trends in education represented by the constructivist theory of learning, teachers still adopt the traditional method of learning, so most researchers believe that the study of teachers' beliefs provides an in-depth look at many of the professional aspects of the teacher.

The desire of the researcher to study this topic was long ago when he was a student, sitting on the chairs of the study, and saw the divergent methods of mathematics teachers in their education and in their view of learning, some of them focused on the information that is installed on the blackboard and transferred to the students' note, and others adore Activities and interact with everyday life. But the researcher did not know from where the mathematics teachers came these beliefs of learning.

The researcher's desire to conduct such a study was compounded by the discrepancies in the results of studies that examined the relationship between the beliefs and teaching practices of mathematics teachers; Some of these studies

emphasize the compatibility of teaching beliefs and practices (Cross, 2009; Gales & Yan, 2001), Some of these findings show inconsistencies between beliefs and practices (Li & Yu, 2010; Sapkova, 2014).

In Jordan, there is no exploration of math teachers' beliefs. Thus, this study aimed at defining math teachers' beliefs about math nature, math teaching and learning, its acquisition sources, and how compatible these beliefs with the teaching practices are. The significance of this study rests in the nature of the problem and the questions it posits. It explores the ideas of the teachers inclusively, not only to identify the beliefs but also attempts to go deep to identify their acquisition sources, which may lead us to a positive direction towards the teachers' ideas.

The significance of this study is also evident in dealing with an essential input of the teaching-learning process, namely, the mathematics teacher. Besides, it tackled three prominent variables in his cognitive system: his beliefs about math nature, it is teaching and learning; sources of its acquisition; and his teaching practices. In processing these variables of the teacher, the results they reveal may give connotations of both the theoretical and applied aspects, to adopt programs to train and qualify the in-service math teachers. These programs enrich the teacher's knowledge and orientations, which may affect his decision inside the classroom. Also, they provide math teacher chances for professional development to more understand the nature of math and its role in forming his beliefs toward its teaching-learning. They further contribute to reshaping his teaching practices, so that they become in line with the calls of the modernization and development movements in math teaching-learning. Furthermore, the study opens a gate for future studies in the same field.

Accordingly, the study problem is defined through answering the following questions:

1) What are the beliefs of math teachers about mathematics, teaching, and learning?
2) Is there a compatibility between the beliefs and teaching practices of mathematics teachers?
3) What are the acquisition sources of math teachers of their beliefs about math nature and about teaching and learning thereof?

2. Methods

The study applied both the quantitative and qualitative methodology so that the quantitative aspect will include categorization and description of the beliefs. The researcher constructed a questionnaire to collect the data and analyze the responses of the participants. As for the qualitative aspect, it includes the observations aiming to search for the teaching practices. Also, it includes interviews, which aim to search for the sources of the teachers' beliefs.

Study Population: One hundred (100) randomly chosen male and female teachers, who teach mathematics to the twelfth grade in the public schools of the Ministry of Education, Jordan, during the 2018/2019 school year.

3. Study Instruments

1) Beliefs Questionnaire: Pursuant to the revisions of the literatures about math nature and math teachers' beliefs, such as (Beswick & Callingham, 2014; Goldin, Epstein, Schorr & Warner, 2011; Kuntze, 2011; Leatham, 2006), the researcher constructed the questionnaire to reveal math teachers' beliefs and their classifications. It consisted of three domains: opinions towards math nature, (8) items, beliefs about teaching math, (9) items, and beliefs about learning math, (9) items. Each item was allocated four choices of the multiple choices, with each alternative was given a grade. The options that are more intimate to the constructivism were: strongly agree with (A) = 4 degrees, agree with (A) = 3 degrees. As for the options that are more intimate to behaviourism, they were: strongly agree with (B) =2 degrees, and agree with (B) = 1 degree. The teacher looks at the two perceptions of each item, and puts (√) in front of the response he most prefers. The following example illustrates the process:

Mathematics could be considered:

A) Group of non-routine situations that require various solution strategies:

B) Group of concepts, rules, and logarithms:

☐ Strongly agree with (A)

☐ Agree with (A)

☐ Strongly agree with (B)

☐ Agree with (B)

As for the content validity, the researcher introduced the questionnaire to several reviewers. They were requested to provide their opinions about the validity of the questionnaire items in terms of clarity and integrity of the language, and ability to measure the three domains of beliefs (math nature, maths teaching, and math learning). In the light of their revision, the researcher carried out the amendments they required.

An exploratory sample of (30) teachers was selected from the study population and from outside the sample, to verify the validity and reliability of the questionnaire. Both were achieved by calculating the Pearson correlation coefficient between the item sub-degree and the total degree of each of the three domains. The results of the analysis showed that (21) items coefficients ranged between 0.61 and 0.78. Meanwhile, (5) items were deleted as follows: one item in math nature domain, two in math teaching domain and two in math learning domain, as their correlation coefficients were low. The reliability was verified through the data collected from the exploratory sample, by obtaining the internal compatibility coefficient of the questionnaire domains and the instrument as a whole, using Cronbach Alpha equations, which ranged between 0.89 and 0.73.

2) Observation Card: The observation card was developed to watch the teachers' pedagogical practices during classroom lessons. The focus was on the ways of teaching the mathematical content, the way the mathematical situations are

represented, as well as focus on the students' thinking, their mistakes, and how to remedy them. The observation card included (35) measurable items with triple grading that expresses the practice degree so that the practice degree will be: high, medium or low, with their digital values, are (3, 2, or 1), respectively.

The validity of the observation card was achieved by introducing to many specialized reviewers, to seek their views about the clarity and integrity of the language and its ability to measure the teachers' practices. Certain adjustments were carried out based on their suggestions. The observation card consisted of (20) items in its final shape. Furthermore, researcher realized the reliability of the observation card by calculating the internal compatibility coefficient using Cronbach alpha equations, which was (0.79), a value that is acceptable for the study purposes (Kawulich, 2005).

The observations were at the rate of three lessons for each teacher.The observer videotaped and wrote down the notes. The researcher was accompanied by one of his colleagues, who got training on the observation card, during the classroom visits of the participants in the study. The card was completed with full independence and objectivity of the observers. After that, the researcher found the compatibility coefficient among their estimates through the use of Cooper equation, which was (0.83), an acceptable value for the study objectives (Kawulich, 2005).

3) Interview to determine the sources of the beliefs: The interview aimed at identifying these sources. The questionnaire is the base through which the interview questions and their axes were built, as they are strongly related to the questionnaire items and axes. For instance, the student's prior knowledge and revealing it by the teacher, learning methods that the teachers follow, their beliefs about the best learning, and other issues such as covering the entire curriculum. Therefore, the researcher, while interviewing the teacher, offers responses of the questionnaire that are pertinent to the teacher, aiming to remind him about his feelings, and link these feelings with their acquisition sources.

The interview comprises (18) questions that investigate the nature of the beliefs and sources of acquisition. It is worth mentioning here that it was a semi-structured interview, which begins with generalities, such is facing the teacher with conclusions about his beliefs, which were detected through his responses to the questions. Then there are general questions about the acquisition sources of the beliefs. After that, the items become more in-depth and specific about the teachers' beliefs on the role of the educational system in acquiring their beliefs, as well as professional colleagueship. In every question, there is a talk about their source of beliefs.

The researcher presented the interview items to the reviewers to seek their views about their clarity and wording. The instrument was applied to ten (10) teachers of the study population, and some unclear items were deleted, which may bear ambiguity for the teachers. Also, items that did not relate to the study objectives were taken out. Accordingly, the interview tool comprised (14) items in its final

shape with focus on: nature and roots of the beliefs, sources of acquiring the beliefs, the educational system, academic study, technological and scientific development, prior knowledge, teaching-learning styles, curriculum coverage, religious belonging, the social environment and self-reflection.

A colleague of the researcher accompanied him after being trained on the interview card, who analyzed five of ten interviews, i.e. 50% of the data. Then the researcher calculated the compatibility rate in the analysis, which amounted to 0.89 in the data analysis, which reflects an acceptable compatibility rate.

4. Results

1) What are the beliefs of math teachers about mathematics, teaching, and learning?

To identify math teachers' beliefs about math nature and their learning-teaching, the teachers' responses to every item in the questionnaire were entered and analyzed. In this concern, the teacher is deemed holding constructivism beliefs if his degree was within (more than 3.25-4). On the other hand, the degree of the teacher, who has a blend of both the constructivism and behavioural beliefs, is within (more than 1.75 – 3.25), and the teacher who has behavioural beliefs has a degree within (1-1.75). The results showed that the teachers hold a blend of swaying opinions, between the constructive and behavioural, about the nature of mathematics, mathematics teaching, and mathematics learning, as shown in Table (1).

Table 1: Means (Ms) and Standard Deviations (SDs) of the Teachers' Beliefs about Mathematics, and Their Teaching-Learning

Domain	M	SD	Belief Classification
Mathematics Nature	2.82	0.52	Blend
Mathematics Teaching	2.52	0.64	Blend
Mathematics Learning	2.75	0.61	Blend

To reveal the contents of the questionnaire items, the researcher took the M of each item according to the responses, as illustrated in Table (2).

Table 2: Means of the Beliefs about Mathematics Nature, Teaching and Learning

No.	Item	M	**Belief Classification**
First: Beliefs about math Nature			
1	Math could be seen as: A) Group of non-routine situations that require various solution strategies. B) Group of concepts, rules and logarithms	2.93	Blend
2	Math could be seen as: A) Renewable knowledge based on discovery and investigation. B) Science characterized by stability, difficult to understand.	2.93	Blend
3	Math could be seen as: A) Mental processes that should be built by the individual. B) Learning products and skills that the student should master.	2.75	Blend
4	Math could be seen as: A) Accumulative science in addition to a network of interlinked thoughts. B) Independent subjects that could be taught in isolation.	2.79	Blend

5	Math could be seen as: A) Communication language in which ideas and modelling are easy to represent. B) Based on the logical judgments, so it is not easy to represent and model.	2.80	Blend
6	Math could be seen as: A) Science and way of thinking have their methodology. B) Art such as music and painting, which needs the talent to be mathematical.	2.82	Blend
7	Most of the students: A) Can practice mathematics through providing them chances to spend efforts and show creative abilities, build ideas and invent methods of solution. B) Cannot practice math because practising needs mathematical talent.	2.72	Blend
Second: Beliefs about math teaching:			
8	When math teacher introduces a new concept, he has to start: A) Discussing the prior thought and concepts the student holds, which facilitate conceptual comprehension. B) Introducing the concept definition and organizing the information around it, regardless of the prior thoughts and concepts the student holds.	2.93	Constructional
9	The most crucial role of the teacher is: A) Motivating the student to discover solutions of the non-routine problems. B) Giving accurate and organized scientific information follow-up of the topics of the schoolbook.	1.60	Behavioural
10	Ways to make students reach concepts mastery levels include: A) Focus on giving new tasks that require the students to link the concepts with each other, to facilitate the conceptual comprehension of the material. B) Bringing worksheets to train the students on the use of the procedures and laws.	2.75	Blend
11	It is better to the teacher to: A) Know his students' thinking, discuss the thought they have, and attempt to change the mistaken understanding with them. B) Focus on displaying and explaining the new ideas.	1.54	Behavioral
12	The role of math teacher during the lessons is provided is: A) Facilitating the students' learning and directing them to build knowledge by their own. B) Transferring the information and presenting it systematically.	2.8	Blend
13	When an unfamiliar effect is presented to the students during math lessons, it is possible to: A) Provide an opportunity for the students to think about the issue of innovating solution strategies to this kind of matter. B) Suggest many strategies for the solution by the teacher, because most students are unable to solve this type of issues.	1.47	Behavioural
14	The main task of a math teacher is: A) To help the students understand the concepts and rules, even if it was at the cost of covering the schoolbook. B) To cover the topics of the schoolbook regardless of the students' understanding of the concepts included in it.	2.72	Blend
Third: Beliefs about math learning			

15	Active math learning comes through: A) Focus on understanding the concepts and concluding the rules to be comprehended by the students in a proper manner. B) Training the students on using the rules to be applied quickly and correctly.	2.93	Constructive
16	For the active math learning to occur, it is inevitable to: A) Provide the students with opportunities to do their utmost in problem-solving and building their own meanings. B) Listen and pay attention to what the teacher explains in the classroom.	2.93	Constructive
17	Math learning is presented by: A) Student's acquisition of new concepts, and linking them with prior knowledge to develop understanding through discussion and dialogue. B) Student's acquisition of new knowledge or behaviour through physical or moral enhancing.	2.75	Blend
18	If the student made a conceptual error during the math lesson, it is better to: A) Discuss the mistaken math concepts to replace them with the correct. B) Ignore and neglect the mistaken concepts and suffice to teach them math ideas effectively.	2.79	Blend
19	Students are taught while they practice a learning activity in mathematics through: A) Cooperative work, as learning through peers leads to increase understanding and to reach an ordinary meaning of the material they are studying. B) Individual learning, because it is better than learning with peers, and strengthens and develops the understanding.	2.8	Blend
20	The student's role during the math lesson is represented by: A) Practising the teaching-learning activities to build the conceptual knowledge and procedural knowledge. B) Listening to math teacher carefully to summarize and learn the rules and procedures by heart.	2.82	Blend
21	Principles of math learning provide that: A) All students could solve the problems if they were given a chance to do their utmost. B) Brilliant students only can solve the problems quickly in the shortest possible time.	2.72	Blend

2) Is there a compatibility between the beliefs and teaching practices of mathematics teachers?

To answer this question, five constructivism teachers and five behavioural were chosen from those who agreed to participate in the study, taking into account the social type and experience factors. In the light of this process, the researcher observed the practices often teachers who completed the questionnaire. Every teacher was provided a code of two-letter abbreviation: the constructive teachers (CT) and the behavioural teachers (BT). Table (3) illustrates the characteristics of the participants.

Table 3: Characteristics of the Participants in the Study

No.	Abbrev.		Years of Service	Academic Degree	Training Courses
1	CT 1	Constructional	10	MA	ICDL, Curricula Development
2	CT 2		8	MA	ICDL Intel, Curricula Development
3	CT 3		7	BA, Mathematics	ICDL, Curricula Development
4	CT 4		13	BA, Mathematics	Intel, ICDL
5	CT 5		8	MA	Intel, ICDL
6	BT 1	Behavioura -	3	BA, Mathematics	ICDL, Curricula Development
7	BT 2		14	BA, Mathematics	Intel, ICDL
8	BT 3		11	BA, Mathematics	ICDL, Curricula Development
9	BT 4		20	BA, Mathematics	None
10	BT 5		22	BA, Mathematics	Intel, ICDL

Male and female teachers' practices were identified through the observation card by finding the means of the teaching practices included in the card. The practices M's fell between (1.02- 2.94), and the following practices topped these M's (2.45-2.94).

- The teacher follows the schoolbook while presenting math ideas;
- The teacher solves math problems on the blackboard and the students write down the solutions;
- The teacher encourages the students to listen and pay attention to what he says;
- He focuses on training the student to learn the rules by heart; and:
- He focuses on the direct presentation type.

It is clear that the above methods are traditional, take up with the teacher based teaching. Students are only listeners and recipients of the information, and focus is only on the knowledge. On the other hand, practices classified as constructional had low degrees within the range (1.02-136), such as:

- The teacher keeps a record of the difficulties the students face during math learning;
- He encourages the students to think loudly to discover reasons of falling in mistakes;
- He concentrates on math ideas representing and modelling to develop conceptual comprehension;
- He posits tasks that encourage thinking; and
- Students practice the ways offered to discover the thoughts.

The analysis showed that (8) male and female teachers (80%) of the sample participants had traditional practices, which focus on the teacher's role, following the schoolbook, focus on the procedures, and positing routine issues. If non-routine problems were displayed, then the teacher solves them. On the other hand, (2) teachers (20%) showed practices that tend to constructivism and problem addressing.

The analysis showed incompatibility between the teachers' beliefs and their practices in the following areas: teaching methods, organizing the classroom environment, the interaction of the teacher-student roles, students' thinking, identifying their errors, and evaluating their learning. For example, the teacher

(CT 2), through her response to item (19), holds social constructional views. She believes in the necessity of the cooperative work among the students to share building math cognition. In item (13), her response showed that when presenting a new mathematical subject, she believes it is essential to help the students link the latest knowledge with prior knowledge. She also believes that students should have a chance to express their opinions on the problem solving, and to discuss them to correct their mistakes and link mathematics with daily life and other sciences.

When we reviewed her teaching practices, we found dominance of the traditional teaching practices. She rarely works towards the students' participation inside the classroom, nor does she allow the cooperative work. Instead, she encourages students to work individually, and focus on the direct presentation method and following the schoolbook to a vast extent. She addresses most of the problems on the blackboard, practices the strict classroom management, focuses on training the female students on procedures to solve the linear equation and learn it by heart, presents the abstract equations, and avoids solving life-related mathematical problems. For instance, in the lesson "solving the two-step linear equation", the equations she presented were such as, solve the equations (***find A value***): 2A=12, 3A+15=18, and so on. All the examples provided here are routine.

3) What are the acquisition sources of math teachers of their beliefs about math nature and about teaching and learning thereof?
The researcher interviewed the teachers who observed their teaching practices to answer this question. He interviewed them as per the following mechanism. In the beginning, the researcher relied on the voice record device, and then he posted the contents of interviews for each teacher separately. The data were studied, coded, and categorized according to the codes or symbols. Therefore, the causes of beliefs at this stage were 135: (60) of the behavioural teachers, and (75) of the constructional teachers.

The researcher searched for specific titles to categorize the data within specific categories, and then searching for types of each group, with literal quotes from the respondents' responses to cite in certain situations. Every category or type had a description, and after that, they were arranged in independent items. Thereby, the researcher could find (15) beliefs of each of the constructional teachers (CT) and (14) beliefs of the behavioural teachers (BT). On the other hand, the shared views between these two categories were (12). Table, (4) show these sources.

Table 4: Math Teachers' Acquisition Sources of Their Beliefs about Math Nature, Teaching and Learning.

No.	Source	Constructional Teachers (n=5)	Behavioural Teachers (n=5)	Total
1	Social Environment	5	5	10
2	Previous Teachers	5	5	10
3	University Study	5	5	10
4	Technology and Media	5	5	10
5	Self-Reflection	5	5	10
6	Personal Experience	5	5	10
7	Professional Colleagueship	5	5	10
8	Scientists	4	5	9
9	Religious Orientations	5	3	8
10	Educational Supervisors	5	4	9
11	Examination System	3	2	4
12	School Principals	3	2	4
13	Experience as Students at Schools	0	5	5
14	School Curricula Structure	0	1	1
15	Free Reading	4	0	4
16	MA Degree	2	0	2
17	Training Courses	3	0	3

Here below are the results of the teachers' beliefs sources.

1- Social Environment: Teachers unanimously agreed that parts of their beliefs are from the social environment where they live. However, these beliefs varied between the (constructivism and behavioural). In this context, (3) teachers said that they acquired behavioural beliefs from the community and the social environment, such as focus on explanation and learning by heart, which was in the beginning of their work as teachers. Teacher (CT 1) referred to this saying: "In the beginnings, I thought that mathematics is a pool of concepts, rules and logarithms, and the more the teacher talks, the more information he communicates are, and this is what I acquired from the community." (BT 4) pointed out to the coercion method in learning when he said: "I acquired beliefs from the social surrounding that the student should be forced to learn and master the mathematical skills. In our communities, if you impose things on them, they will obey, but if you give them freedom, they lax.

2- Previous Teachers: All the sample participants said that they acquired many of their beliefs from the teachers they taught them in the past throughout all the educational stages. Four teachers said that their previous teachers were a major source of their beliefs, especially in the beginning of their work as teachers. Teacher (CT 2) said: "I was influenced by those who taught me in the beginning, and maybe still now, but not in everything." In the same context, as for the behavioral teachers, the Teacher (BT 4) said: "Surely, I learned from a math teacher who taught me in the primary grades, firstly, and another professor in the university who taught me the teaching method. In other words, clarifying the points in detail on the blackboard and then asking the students to continue the explanation after the lesson. I think that learning is communicated by introducing the information in a correct and organized manner."

3-University Study: The teachers, unanimously, said that their university study offered them beliefs about mathematics and it's teaching-learning; and the influence of the university varied among the surveyed teachers. Teacher (CT 1) and Teacher (CT 4) acquired the cooperative learning and discussion methods; especially they apply them in the university with their instructors before applying to their students. On the other hand, both Teacher (BT 4) and Teacher (BT 1) acquired behavioural beliefs, such as a focus on calmness and classroom discipline.

4- Technology and Media: The teachers said that they acquired their beliefs from the technological development as well as multiple media. They shared the belief that technology and media made them believe that students have ideas about mathematics. They search on the internet and other technological sources. Teacher (CT 4) said: "The student may obtain from the internet information that you do not know." By talk about how to identify the students' concepts and knowledge in mathematics, Teacher (BT 1) sees that the use of the exploratory method may help in identifying the students' knowledge, and he acquired this from technology. He referred to this and said: "Through technology, video, and pictures displaying, I utilize an exploratory method to identify the students' knowledge. For sure, students want a change; I mean the internet and Facebook generation will feel tediousness when you "talk" through chalk and silent image."

5- Self-Reflection: All the sample participants emphasized that they use self-reflection through thinking of what they are doing in teaching, and asking themselves questions about their teaching methods. They acquire many beliefs through their self-revision, such as belief in the need to activate the student's role in sharing the learning process. Teacher (CT 4) talked about his early math teaching and the development of his beliefs about the important knowledge in learning. He said: "In the beginnings of my work, I focused on the theoretical aspect, i.e. learning by heart. Gradually, I shifted towards the students' participation in the lesson, and encouraged them to search and investigate, through my reflection on my teaching methods and comparing them with former and current ones."

6-Personal Experience in Teaching: The teachers said that they acquired their beliefs through the experience they passed, including mathematics, which are mental processes that should be built by the individual. They added that students have thoughts and experience about mathematics. Teacher (CT 1) said: "Before this experience, I thought that when I ask the student to define something, he will give the definition. But, when I examine him, he gets a good grade, which means that he is learning by heart. Now, since the beginning of the class, I ask questions to probe whether the student has information. I began focusing on my student in the classroom to find out whether he masters the required skills, clarifies the concept and uses it in his daily life."

Behavioural teachers indicated that they acquired certain beliefs about learning through experience and over their teaching period. For instance, they belief that

mathematics is a science characterized by stability, not easily understood. Therefore, students should be trained to focus on using the rules and concentrate on training, as they have no prior knowledge about mathematics.

7- Professional Colleague-ship: It seems that math teachers acquired a part of their beliefs about mathematics and math learning-teaching from their professional colleagueship with other teachers. Especially at the beginning of their work as teachers, when they did not possess sufficient teaching experience. However, some of the beliefs they acquired, in their entirety, were behavioural, others were constructional. Teacher (CT 1) said in this concern: "I used to ask another teacher who is more experienced than me (11 years of experience) at school. Later I discover that his teaching methods are traditional". Their colleagues influenced most of the behavioural teachers through their experiences and knowledge. Teachers (BT 1 and BT 2) see that they acquired behavioural beliefs, such as the focus on the simple knowledge and transfer and display of the information in an organized manner.

8- Scientists: Four teachers said that they acquired their beliefs from the biographies and experiences of scientists and scholars. However, one teacher did not acquire any of his beliefs from this source, namely Teacher (CT 4). In this connection, Teacher CT 1) said that he acquired his beliefs from Paulo Freire's writings. He said: "Paulo Freire's writing changed my understanding of the teacher's job in his essay, "The Seventh Letter". The works of scholars and experts influenced all the behavioural teachers. The Teacher (BT 1) acquired his beliefs from some Muslim scholars and scientists, such as Al-Razi, by using sensible things in teaching. He added: "He often uses the perceived, watch-based work style."

9- Religious Orientations: Teachers said that they acquired their constructivism beliefs from their religious belonging, which were clear in the importance of dialogue and discussion, the necessity of the cooperative learning, and listening to the students. Religion encourages cooperation in all the wakes of life, as well as decision-making, in which the male and female teachers believe.

10- Educational Supervisors: One third of the teachers indicated that they acquired part of their beliefs from their supervisors. They knew from their supervisors that renewable knowledge is built on discovery, investigation, dialogue, and importance of activating the student's role. Teacher (CT 5) said: "I acquired from my supervisors the importance of the knowledge of the content, and that mathematics is a network of interconnected ideas, in addition to supporting the students' role and discussions with them."

11- Examination System of the Ministry of Education: Two of the constructional teachers and two of the behavioural said that they acquired behavioural beliefs from this system, such as the focus on covering the curriculum in the first place, and preferring it over dialogue and discussion. Therefore, in their examinations, they focus on measuring the "amount" of knowledge the student has, and how far he can retrieve it. They justify this by that the examinations provided by the

Ministry of Education stress on covering the curriculum from A to Z. Thereby, teachers are compelled to cover the curriculum, not because they believe in doing so, but because it is something beyond their wills.

12- School Principals: The interviews revealed that one- third of the constructivism teachers gained their beliefs from the school principals, such as caring for the students. Teacher (CT 1) said: "I acquired from some principals beliefs of caring for the students and establishing dialogues with them always." In this connection, Teacher (BT 1) said that his principal was the best model in teaching. He said: "I have a great example, my principal; it is not easy to describe this gentleman; he writes the title of the lesson, explains it, and then illustrates the basic points one by one."

13- Their Experience as Students at Schools: All the behavioural teachers said that they acquired a part of their beliefs from their studies at schools while sitting in their seats. Teacher (BT 1) referred to this and said: "I encourage the group work. I have my views in dealing with the students and teaching since I was a child until I became a teacher."

14- Curricula Structure: One teacher only acquired his beliefs about math learning from the curriculum nature, as he believes in the importance of training at the expense of conclusion. Teacher (BT 4) said in this regard: "Most often, I focus on training because this is the method of the curricula per se." Meanwhile, none of the constructional teachers said that he acquired his beliefs from the curricula structure.

15- Free Reading: Free reading had its role in the constructivism teachers' acquisition of their beliefs, as most of them indicated that they acquired their beliefs through it. Teacher (CT 2) said: "I have a book in math teaching, and I still read from it till now."

16- MA Study: Three constructivism teachers acquired their beliefs from MA studies. In this concern, Teacher (CT 1) said that he acquired them through MA studies, when he studied the social constructional thought in learning. He said: "I believe in the social constructivism, because the student doesn't come to the classroom without ideas about the topic to learn, especially with the technological development. He may also hear from his brothers or siblings about the topic." On the other hand, none of the behavioural teachers said that he acquired his beliefs from MA studies.

17- Training Courses: Training courses played a significant role in the constructivism teachers' acquisition of their beliefs. Teacher (CT 1) acquired beliefs about the importance of the prior knowledge, and the need for revealing it. In this connection, he said: "Before I received my training, it never came to my mind that the student has information, which I can discover."

5. Discussion and Conclusions

The results showed that teachers who hold a blend of the constructional and behavioural beliefs are the largest portion of math teachers. It may be ascribed to that teachers do not hold a positive thought about mathematics and their teaching-learning. Furthermore, the traditional method was the predominant in most of the teachers' teaching practices, which does not keep up with the modern view of math teaching-learning. Most of the teachers were focusing on fragmenting and disconnecting math subject, coining the behavioural outcomes, focusing on the individual work, use of the direct presentation method, and dialogue and discussion sometimes. They further concentrated on the procedural knowledge with attempts by some of them to link the conceptual with procedural knowledge. The teacher-based teaching model was dominant, with chances given to the students' participation at times; and teachers focus on adhering to the schoolbook, learning the rules by heart, and training the students on the procedures. There were agreements of two teachers (20% of the sample) between their beliefs and teaching practices. The agreement may be attributed to the motivation that they have toward professional developed, as they voluntarily attended the courses held by the Ministry of Education. On the other hand, there are disagreements among (8) male and female teachers (80%) about their beliefs and practices. This result is in line with many studies, such as (Arı, Tunçer & Demir, (2016; Brendefur & Carney, 2016; Beswick, 2012; Hilal & Ucar, 2010; Leatham, 2006; Polly, McGee, Wang, Lamber, Pugalee & Johnson, 2013; Polly, Neale & Pugalee, 2014; Zakaria & Maat, 2012).

The study showed that both the constructivism and behavioural teachers acquired some of their beliefs about learning from their former teachers, current colleagues, personal experience, university studies, social environment, and the technological development that the modern age witnesses. Besides, their self-reflection to evaluate their previous achievement and performance every school year helped in this regard. The study further showed that the teachers have non-structural beliefs about learning. But, when they attempt to apply their views in the field, they discover that what they believe is incompatible with the reality of the educational system. Furthermore they find a wide gap between the two issues, which make them sometimes give up their beliefs to realize the best interest of the student.

6. Recommendations

1) Designing programs to prepare mathematics teachers for different levels of education so as to build a balanced and coherent system of beliefs towards the nature of mathematics, teaching, and learning.
2) Organize training courses aimed at the professional development of in-service mathematics teachers so as to enhance their constructive beliefs and change traditional beliefs and practices.
3) the need to review the teacher training programs to make them based on more modern foundations based on dialogue, discussion, critical thinking, and self-reflection; and to employ modern educational theories in the educational process, so that teachers gain constructive beliefs.

4) Promote the idea of cooperation between teachers through mutual visits, group meetings and research lessons accompanied by dialogue, because the results of the study indicate that the constructive and behavioral teachers have acquired a mix of beliefs from their colleagues.

7. Study Limitations and Delimitations

1) This study was limited to mathematics teachers teaching 12th grade for the academic year 2018/2019, (10) teachers of the participants of the study agreed that the researcher to attend classes, and video recording. For these classes, and teachers who were not ready to participate that Excessive adherence to other educational activities is the main reasons for non-participation.
2) Since the questionnaire of beliefs and the observation card is prepared by the researcher, the results of the study may be influenced by the reliability and reliability of these tools.
3) The beliefs questionnaire was limited to three areas: mathematics teachers' beliefs about the nature of mathematics, and teaching and learning mathematics. They do not cover all areas of mathematics teachers' beliefs

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