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Constructivist Learning and the Law of Sines in Advanced 10th Grade Geometry Textbooks in Vietnam

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Abstract. The law of Sines is the law that appears much in mathematics in particular and in science in general. The law of Sines allows us to calculate the length of the remaining sides and angles when we know three of the six elements of the side or angle of a triangle. This law, along with the law of Cosines, are the first two laws, the most important when one wants to build a trigonometric system. In Vietnam, they teach the law of Sines in Geometry 10 program, including the Basic and Advanced sections. Constructivist learning is one of the theories of the teaching process based on Piaget's psychology generating awareness and Vygotsky's operation theory. There are many different views on constructivist learning. All of these points of view agree that this is a positive teaching method, promoting the internal strengths of learners. In other words, this is a learner-centered teaching method. Although the law of the Sines function is important, there are not many documents that mention how to teach the law of Sines effectively. The paper examines the combination of the constructivist learning method and the law of Sines as well as the relationship between mathematics and other fields expressed through this law.

Keywords: Law of Sines; Constructivist learning; the real problem; Informatics; Physics

1. Introduction

The content of the constructivist theory has existed for a long time. According to Husen, T., & Postlethwaite, TN (1989), the content of the constructivist theory appeared in Giambattista Vico's De antiquissima Italorum Sapientia in 1710. He thought that "to know something means to know what parts it is made of and how they have been put together." In modern psychology, Mark Baldwin (1861-1934) and Jean Piaget (1896-1980) first developed the concept of cognitive construction. Among the pioneers of the development of the constructivist theory, Jean Piaget is one of the most knowledgeable about the constructivist theory. According to Jean Piaget, the cognitive structure develops gradually in the process of a subject adapting to the environment. That adaptation starts at birth as a result of natural physiological development and experience of environmental exposure. Children transform and develop structures to function, think, and perceive the world, so they become more and more sophisticated with age. (Tran, T. M. L, 2012)

This cognitive structure develops according to a dual process, the assimilation process, and the accommodation process (Tran, T. M. L, 2012). The process of assimilation is the process of transforming new knowledge by the existing cognitive structure, so that the subject can rely on old knowledge to solve new situations. It is necessary to fully exploit the students' existing knowledge and experience related to the knowledge to teach as the basis for creating new knowledge. In particular, it is necessary to identify the "input" as the knowledge and experience that students know, analyze the "black box" as thinking manipulations such as: analysis and synthesis, generalization, analogy, systematizing knowledge to build a process of organizing learning activities for students. The process of accommodation is the process of transforming new knowledge when it is different from the existing cognitive structure, forcing the subject to change the cognitive structure to suit the new knowledge. In doing so, we need to exploit misconceptions (or inadequacies) of students as a basis for designing learning activities, thereby building a process of organizing learning activities for students. Thus, the assimilation process leads to the growth of old knowledge structures, which is the process of dealing with new information of the environment in the form of pre-existing thinking, while the process of accommodation creates the development of new knowledge structure and the process which the subject transforms the previous cognitive structure according to the interactions with the environment. (Tran, T. M. L, 2012)

The second person that we cannot help but mention when studying Constructivism is Vygotsky. He is regarded as the father of the zone of proximal development (ZPD) theory. According to Vygotsky, teaching is the origin of newness in development and unity with the old. Teaching always needs to be ahead of development (teaching pulls development along with you). The teaching process is carried out through the individual psychological characteristics of students. Vygotsky acknowledged that "Teaching, in one way or another, must be appropriate to the child's developmental level, which is a fact discovered by experience and tested over and over again, undeniable. ”. (Vu, T. N, 2014)

Vygotsky said that the child development process often takes place at two levels of the present level and the zone of proximal development. The present level is the level at which the psychological functions have reached maturity, and in the zone of proximal development, the psychological functions are maturing but not yet mature. In practice, the current level of performance is expressed by children independently solving tasks without any help from the outside, and the zone of proximal development is shown in the situation when children complete the task when there are cooperation and help from others, and they cannot do it themselves. Thus, two levels of child development represent two degrees of maturity at different times. At the same time, they are always in motion: the zone of proximal development today will become the current level tomorrow, and the new zone of proximal development will appear. (Tran, T. M. L, 2012)

Many people have studied constructivist learning. For example, Jones, M. G., & Brader-Araje, L (2002), Husen, T., & Postlethwaite, T. N (1989) studied the impact of constructivism on Education which are language, discourse, and meaning. Olusegun, B. D (2015), Sjoberg, S (2007) referred to a paradigm for teaching and learning of constructivist learning theory. Matinez-Delgado, A (2000) gave to the radical constructivism. Suhendi, A., & Purwano, P (2018). Koohang, A., Riley., L., & Smith, T (2009) determined e-learning and contructivism. Mogashoa, T (2014) researched the applicability of the constructivist theory in qualitative educational research. Ramsook, L., & Thomas, M (2016) referred to constructivist-linking theory with the practice among pre-service teachers at the University of Trinidad and Tobago. Ayaz, M. F., & Sekerci, H (2015), Maheshwari, G., & Thomas, S (2017), Bhattacharjee, J (2015), Le, T. L. H., Luu, T. T., & Nguyen, T. L. A (2016), Doan, T. K. N., & Hoang, T. M. T (2016) explored the effects of the constructivist learning approach. Nguyen, T. T., & Dang, T. X (2013) used the Google Sites tool to set up a website to boost students' self-study ability on the constructivist theory.

Nguyen, D. N., & Manyvanh Inthavorigsa (2018) talked about teaching the constructivist topic "primitive - integral" in high schools of the People's Democratic Republic of Laos. Truong, T. H (2011) discussed testing and assessing the formation process of Vietnamese knowledge in High School from the perspective of the constructivist theory. Nguyen, T. H (2013) knew how to apply the constructivist theory in teaching Education at pedagogical schools. Tran, V. D (2015) presented a constructivist theory that underpins collaborative learning. Pham, V. H (2016) referred to the practical lesson design skills of the constructivist theory of elementary pedagogical students. Nguyen, T. T., Hoang, T. P., & Tran Trung Ninh (2014) studied the development of the ability to apply knowledge into practice for students by applying the constructivist theory to teaching chemistry 10.

Although there have been many studies on teaching with the constructivist theory and teaching the law of Sines, there have not been any researches on teaching the law of Sines by the constructivist theory. In this paper, we study how to teach the law of Sines according to the constructivist theory in Vietnam in this article.

2. Content

2.1. Teaching according to the constructivist theory

Currently, there are many different views on the constructivist theory. The constructivist theory states that the learners' knowledge, skills, and competences are not an "empty box" for teachers to teach what they want to. The teacher only imparts knowledge to the learners based on the existing knowledge base on the learners' experience. Learners only acquire when connecting new knowledge and their life experiences. The close relationship between new and old knowledge is systematically arranged, then the new knowledge is valuable to use and remember longer.

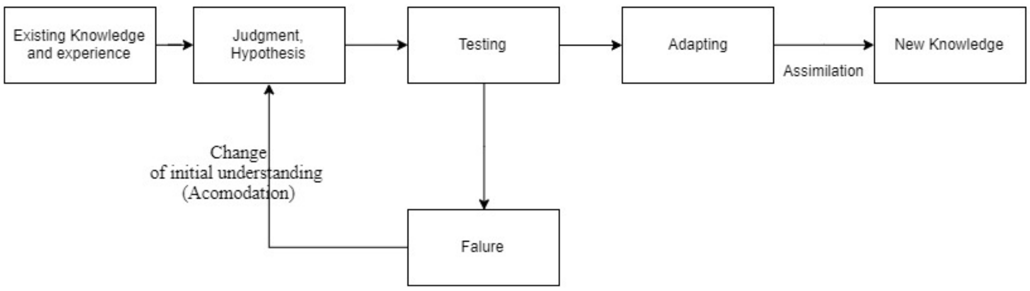
"*Constructivism is not a theory about teaching… it is a theory about knowledge, and learning…the theory defines knowledge as temporary, development, socially and culturally mediated and thus, non-object*" (Brooks & Brooks, 1993, p. vii)

"*Constructivist allege that it is we ho constitute or construct, based on our theorizing or experience, the allegedly observable items postulated in our theories.*" (Nola, 1998)

"*Knowledge, no matter how it is defined, is in the heads of person and that the thinking subject has no alternative but to construct what he or she knows based on of his or her own experience.*" (Glasersfeld, 1992)

From the above points of view, we propose the concept of the constructivist theory as follows: Constructivism is the process by which subjects perform assimilation and accommodation activities, to establish new knowledge based old knowledge. This process of establishment is not a process of mechanical establishment. Still, the process of a subject perceiving, eliminating outdated, inappropriate, and inheriting the core, nature, correct, thereby adjusting, developing experience and available capacity to receive knowledge and build new knowledge for himself or herself. The process of establishment is not only the discovery but also the explanation and the structure of the new experience to receive, or it is the process of adaptation and evolution. This process is both personal and public.

2.2. The process of learning according to the constructivist theory

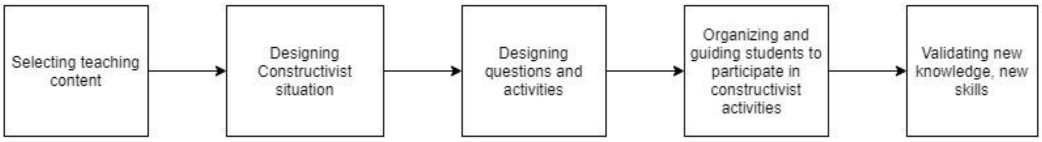


(Bui, V. N, 2009)

3. An Illustrative example of applying the constructivist theory to teaching the law of Sines

For a constructivist teaching method to succeed, a teacher must create learning situations that stimulate students' interest in learning. That understanding must be received by learners in a positive, proactive, and creative manner. Learners must discover knowledge by themselves, not passively learn from the environment. Learners both act as instructors, leading the problem to help students confirm the correctness. Depending on the specific teaching content, specific teaching subjects, teachers give students different learning tasks. Students interact with each other to create an atmosphere, both personal and socia,l in the classroom.

Bui, V. N. (2009) provided the following steps for designing and implementing a constructivist teaching lesson:



Here we will illustrate how to teach a math problem through the law of Sines.

- Step 1. Select teaching content

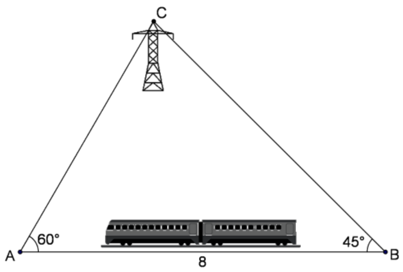
Teacher: Our teaching content is the law of Sines.

- Step 2. Design a constructivist situation

Teacher: Please see the following problem.

Example 1

A person is sitting on a train from station A to station B. When the train was at station A, through binoculars, he saw a high-voltage pole C. The direction of his view to the high-voltage pole created with the path of the train at an angle of 60^o. When the train was at station B, the person looking back could still see the tall voltage column C, the view from that person to the high voltage pole created in the opposite direction of the train's path at an angle of 45^o. Given that the railway section that connects Station A with Station B is 8 km long. What is the distance from station A to tower C? (Doan, Q., et al. 2019)



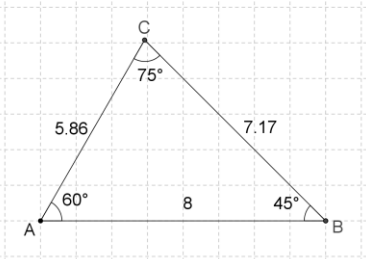
Teacher: To find the distance from A to C tower we need to solve the problem of finding the side AC when we know the edge AB; angle $C = 180^\circ - 60^\circ - 45^\circ = 75^\circ$; angle $B = 45^o$.

- Steps 3 & 4. Design questions, activities and guide students to participate in the constructivism.

Teacher: Draw the triangle ABC in example 1, measure the lengths of the edges, the angles, and then fill in the following table:

A	a	$\frac{a}{\sin A}$	B	b	$\frac{b}{\sin B}$	C	c	$\frac{c}{\sin C}$

Students: We do not know how to draw and measure shapes on GeoGebra software.
Teacher: I will draw and measure BC on GeoGebra software as follows.
- Draw a line AB with a length of 8.
- Draw the rays AC and BC so that $BAC = 60^{\circ}$; $ABC = 45^{\circ}$.
- Hide unnecessary routes.
- Measure angle A, sides AC and BC.



We have the following table:

A	a	$\frac{a}{\sin A}$	B	b	$\frac{b}{\sin B}$	C	c	$\frac{c}{\sin C}$
60°	7.17	8.28	45°	5.86	8.28	75°	8	8.28

Teacher: What do you think about the quantities $\frac{a}{\sin A}, \frac{b}{\sin B}, \frac{c}{\sin C}$?
Students: These quantities are equal.
Teacher: Can you show that the triangle ABC has
$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$?

Student: We haven't found a way to prove it yet.
Teacher: If $\frac{a}{\sin A} = \frac{b}{\sin B}$, then, $a^2 \sin^2 B$ is equal to $b^2 \sin^2 A$ or not?
Students: Yes.

Teacher: If $a^2 \sin^2 B = b^2 \sin^2 A$, then, $a^2 - a^2 \sin^2 B - (b^2 - b^2 \sin^2 A)$ is equal to $a^2 - b^2$?
Students: Yes.
Teacher: Put the common factor of the expression
$a^2 - b^2 = a^2 - a^2 \sin^2 B - (b^2 - b^2 \sin^2 A)$?

Student:

$$a^2 - b^2 = a^2 - a^2\sin^2 B - (b^2 - b^2\sin^2 A)$$
$$= a^2(1-\sin^2 B) - b^2(1-\sin^2 A)$$
$$= a^2\cos^2 B - b^2\cos^2 A$$
$$= (a\cos B - b\cos A)(a\cos B + b\cos A).$$

Teacher: Use the law of Cosines to calculate $a\cos B - b\cos A$; $a\cos B + b\cos A$?

Student: I calculate the following:

$$a\cos B - b\cos A = \frac{c^2 + a^2 - b^2}{2c} - \frac{b^2 + c^2 - a^2}{2c} = \frac{a^2 - b^2}{c};$$
$$a\cos B + b\cos A = \frac{c^2 + a^2 - b^2}{2c} + \frac{b^2 + c^2 - a^2}{2c} = c.$$

Teacher: So $(a\cos B - b\cos A)(a\cos B + b\cos A) = ?$

Student: $(a\cos B - b\cos A)(a\cos B + b\cos A) = \dfrac{a^2 - b^2}{c}.c = a^2 - b^2.$

Teacher: Please deduce the solution.

Student: Suppose that a, b, c are three sides and A, B, C are three angles respectively with three sides a, b, c. Applying the law of Cosines, we have:

$$a^2 = b^2 + c^2 - 2bc\cos A \Rightarrow b\cos A = \frac{b^2 + c^2 - a^2}{2c} \text{ (1).}$$
$$b^2 = a^2 + c^2 - 2ac\cos B \Rightarrow a\cos B = \frac{a^2 + c^2 - b^2}{2c} \text{ (2).}$$

From (1) and (2) we have:

$$a\cos B - b\cos A = \frac{c^2 + a^2 - b^2}{2c} - \frac{b^2 + c^2 - a^2}{2c} = \frac{a^2 - b^2}{c};$$
$$a\cos B + b\cos A = \frac{c^2 + a^2 - b^2}{2c} + \frac{b^2 + c^2 - a^2}{2c} = c.$$

So $(a\cos B - b\cos A)(a\cos B + b\cos A) = \dfrac{a^2 - b^2}{c}.c = a^2 - b^2.$

Or: $$a^2\cos^2 B - b^2\cos^2 = a^2(1-\sin^2 B) - b^2(1-\sin^2 A)$$
$$= a^2 - b^2 + b^2\sin^2 A - a^2\sin^2 B = a^2 - b^2.$$

Therefore: $a^2\sin^2 B = b^2\sin^2 A$. Inferred: $a\sin B = b\sin A \Rightarrow \dfrac{a}{\sin A} = \dfrac{b}{\sin B}$.

Similarly, we have the formula: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.

- Step 5. Validation, consolidation of new knowledge, new skills.

Teacher: Say the law of Sines:

Student: For every triangle ABC, we have: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.

Teacher: Let's return to example 1. Do you know the length of the edge AB?

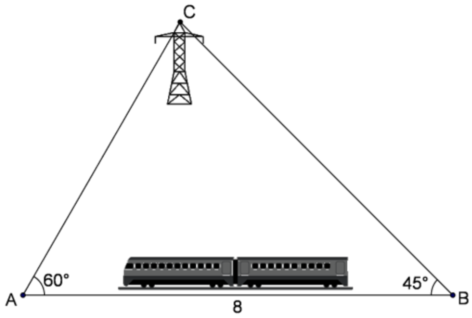
Student: AB = 8

Teacher: What about angles C, B?

Student: $C = 180^o - 60^o - 45^o = 75^o$; $B = 45^o$.

Teacher: Please use the law of Sines to prove the problem.

Student: I demonstrate the problem as follows:



Take a look at the triangle ABC. We have

$C = 180^o - (60^o + 45^o) = 75^o$.

Applying the Law of Sines to the triangle ABC, we get $\frac{b}{\sin B} = \frac{c}{\sin C}$.

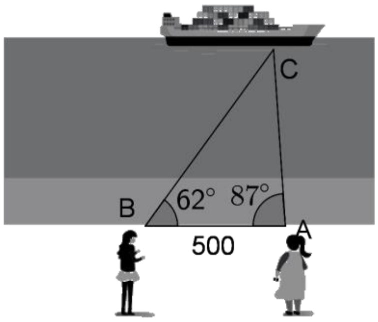
Inferred $b = 8 . \frac{\sin 45^o}{\sin 75^o} \approx 6\ (km)$.

So the distance from Station A to Tower C is approximately 6km.

Teacher: Please apply the law of Sines to solve the following similar problem:

Example 2

In the figure, there is a ship moored at position C on the sea, and there are two people at observation positions A and B 500 meters apart. They measured CAB angle of 87^o and CBA angle of 62^o. Calculate the distances AC and BC (Doan, Q., et al. 2019)



Student: I solve the problem as follows:

We have $C = 180^o - 87^o - 62^o = 31^o$.

Applying the Law of Sines to the triangle ABC, we get $\frac{BC}{\sin A} = \frac{AC}{\sin B} = \frac{AB}{\sin C}$.

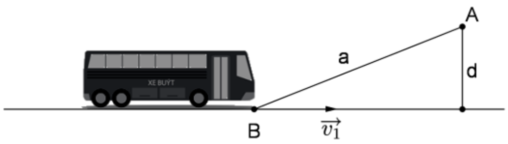
Or $\frac{500}{\sin 31^o} = \frac{AC}{\sin 62^o} = \frac{BC}{\sin 87^o}$. So $AC = \frac{500.\sin 62^o}{\sin 31^o} \approx 857\ (m)$.

Similarly: $BC = \frac{500.\sin 87^o}{\sin 31^o} \approx 969\ (m)$.

Teacher: Please solve the same problem in physics as follows:

Example 3

A bus named B moves steadily with a speed of $v_1 = 54km/h$. A passenger named A is at a distance from the bus $a = 400m$, and at a distance from $d = 80m$, to take the bus. Which direction and what minimum speed does he have to run to catch the bus? (Bui, Q. H., et al. 2003)



Teacher: Call C the position of the car and the person meeting; the velocity the person runs to meet the bus is v_2; Δt is the time from the start of the run until the moment when he or she meets the bus. Can you show AC, BC in v_1, v_2 and Δt?

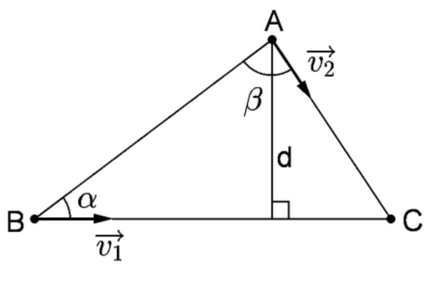
Student: $AC = v_2.\Delta t;\ BC = v_1.\Delta t.$

Teacher: If we apply the law of Sines to the triangle ABC, what do we get?

Student: $\frac{v_2 \Delta t}{\sin \alpha} = \frac{v_1 \Delta t}{\sin \beta} \Rightarrow v_2 = \frac{\sin \alpha}{\sin \beta}.v_1$. Inferred v_2 has minimum value with $\beta = 90^o;\ (v_2)_{\min} = \sin \alpha . v_1 = \frac{d}{a} v_1 = 10{,}8\ (km/h)$.

Teacher: Please complete the solution to the problem.

Student:



Suppose that C is the location of the meeting. $AC = v_2.\Delta t;\ BC = v_1.\Delta t.$

Apply the law of Sines to the triangle ABC:

$\frac{v_2 \Delta t}{\sin \alpha} = \frac{v_1 \Delta t}{\sin \beta} \Rightarrow v_2 = \frac{\sin \alpha}{\sin \beta}.v_1.$

Inferred v_2 has the minimum value with $\beta = 90^o$;

$$(v_2)_{\min} = \sin\alpha . v_1 = \frac{d}{a} v_1 = 10,8\ (km/h).$$

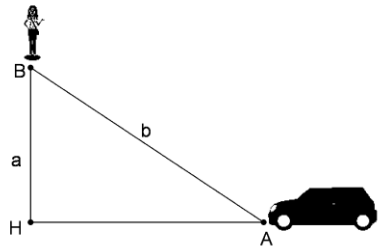
Teacher: The following is another example that illustrates the law of Sines in physics similar to example 3.

Example 4

A taxi moves straightly on the road with a speed of $v_1 = 16\ m/s$. A passenger stands 60 meters from the road. This person saw the taxi at a time when the vehicle was about b = 400 meters away.

a. Which direction must the person run to get to the way at the same time or before the cab gets there? Given that the average human speed is $v_2 = 4\ m/s$.

b. If he wants to meet the vehicle at the lowest speed, which direction does he have to run? What is the smallest velocity? (Le, V. V, 2018)

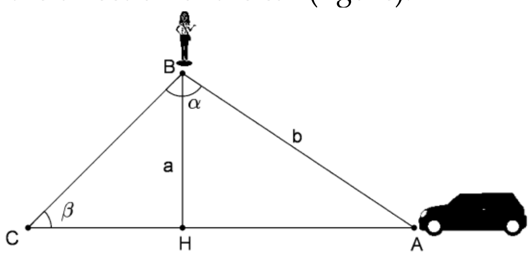


Teacher: Please use the solution shown in example 3 to find the answer to this problem.

Student: I solve the problem as follows.

a) The direction of the person running to meet the bus.

Assume α is an angle formed by the direction of the person to the vehicle and the path the a person must run; β is the angle formed by the direction the person running and the direction of the car (figure).



Applying the law of Sines to the triangle ABC, we have:

$$\frac{\sin\alpha}{AC} = \frac{\sin\beta}{AB} \Rightarrow \sin\alpha = \frac{AC}{AB}\sin\beta$$

with

$$AB = b;\ AC = v_1 t_1;\ BC = v_2 t_2;\ \sin\beta = \frac{a}{BC} = \frac{a}{v_2 t_2} \Rightarrow \sin\alpha = \frac{v_1 t_1}{b}.\frac{a}{v_2 t_2}\ (1)$$

If the person arrives before the vehicle does: $t_2 \le t_1$ (2)

Inferred $\sin\alpha = \frac{16t_1}{400} \cdot \frac{60}{4t_2} = \frac{0,6t_1}{t_2}$ (1')

Inferred $\sin\alpha \geq 0,6 \Rightarrow 36^o45' \leq \alpha \leq 143^o15'$.

So to meet the bus he has to run in the direction that creates with the path from the person to the car at an angle of $36^o45'$ to $143^o15'$.

b) The minimum speed for the person to meet the vehicle.

For the person to meet the vehicle at the lowest speed $t_2 = t_1$ and $\sin\alpha = 1$.

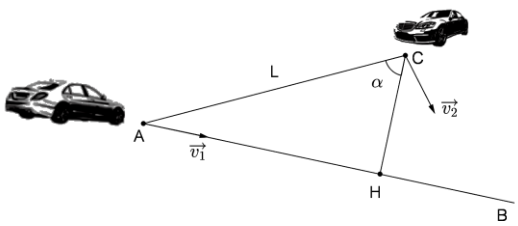
Inferred $\frac{v_1}{b} \cdot \frac{a}{v_2} = 1 \Rightarrow v_2 = v_{2_{\min}} = \frac{a}{b} v_1 = \frac{60}{400}.16 = 2,4\ (m/s)$.

So the minimum running speed for the person to meet the vehicle is $v_{2_{\min}} = 2,4\ (m/s)$, and the direction then is perpendicular to the direction where he or she sees the car.

Teacher: Another problem similar to example 4 is the following:

Example 5

The car I starts from point A running on straight line AB with a velocity of v_1. At the same time, car II starts at point C, which is a segment L away from A, with a speed of v_2 to reach car I. Given that the segment AC creates with CH line at an angle α.

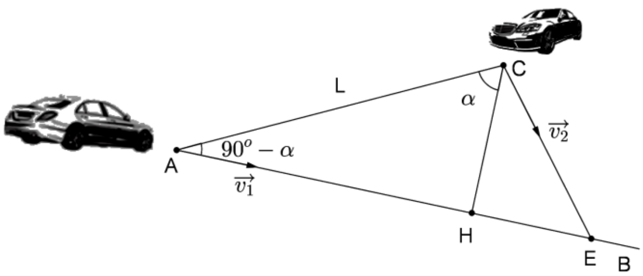


a) Which way does car II have to travel to meet car I and how long does it take to reach car I?

b) Find conditions for two vehicles to meet at H (Le, V. V, 2018)

Teacher: Please solve this problem.

Student: I've solved the problem as follows.



a) Time for two vehicles to meet is:

Suppose two vehicles meet at point E after time t.

We have: $AE = v_1 t$ and $CE = v_2 t$. Inferred $\frac{v_1}{v_2} = \frac{AE}{CE}$ (1)

Applying the law of Sines to the triangle ACE, we have:

$$\frac{\sin(90^o - \alpha)}{CE} = \frac{\sin\beta}{AE} \quad (2)$$

From (1) and (2) we have: $v_1 \sin(90^o - \alpha) = v_2 \sin\beta \Rightarrow v_1 \cos\alpha = v_2 \sin\beta$ (*)

So car II must go in the direction of CE and create with AC an angle β with $\sin\beta = \frac{(v_1 \cos\alpha)}{v_2}$.

Condition: $v_1 \cos\alpha \le v_2$.

The projections of $\overrightarrow{v_1}$ and $\overrightarrow{v_2}$ down to AC are $v_1 \sin\alpha$ and $v_2 \cos\beta$.

Time t needed for vehicle II to meet vehicle I is: $t = \frac{L}{v_1 \sin\alpha + v_2 \cos\beta}$.

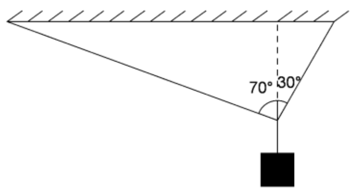
From (*), we infer $v_2 = \frac{v_1 \cos\alpha}{v_1 \cos(\alpha - \beta)}$, we get $t = \frac{L \sin\beta}{v_1 \cos(\alpha - \beta)}$.

b) For two cars to meet at H, we must have $\beta = \alpha$, then from (*) we conclude $\tan\alpha = \frac{v_1}{v_2}$ and $t = \frac{L\sin\alpha}{v_1} = \frac{L\cos\alpha}{v_2}$.

Teacher: Please continue to come to the law of Sines through the following practical problem:

Example 6

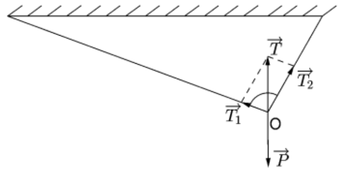
An object with mass m = 2kg is hung by two inelastic ropes that intersect with the vertical direction at angles of 70^o and 30^o respectively (figure).



Determine the tension of each rope. Take $g = 10\ m/s^2$. (Bui, Q. H., et al. 2003)

Teacher: The object with mass m hanging at O is in equilibrium. The equilibrium O point is affected by three forces: $\overrightarrow{T_1}$, $\overrightarrow{T_2}$ tension forces and weight $\overrightarrow{P}$ of object m. The forces of $\overrightarrow{T_1}$, $\overrightarrow{T_2}$ and $\overrightarrow{T}$ are in balance with $\overrightarrow{P}$. Can you use the law of Sines to solve the problem?

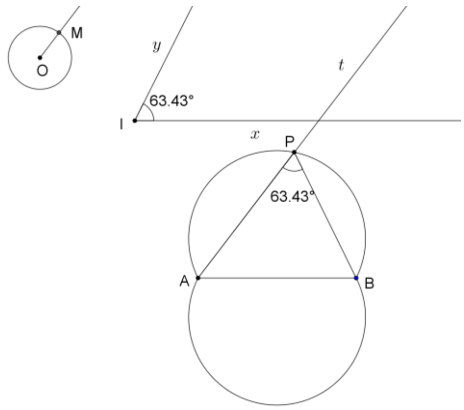
Student: I have solved the problem as follows:
The object with mass m hangs at O in equilibrium. The equilibrium O point is affected by 3 forces: $\overrightarrow{T_1}, \overrightarrow{T_2}$ tension forces and weight $\vec{P}$ of object m. The synergies $\overrightarrow{T_1}, \overrightarrow{T_2}$ and $\vec{T}$ are in balance with P.



Applying the law of Sines the triangle, we have:
$$\frac{T}{\sin 80^o} = \frac{T_1}{\sin 30^o} = \frac{T_2}{\sin 70^o},$$
here $T = P$, so:
$$T_1 = P.\frac{\sin 30^o}{\sin 80^o} = 2.10.\frac{\sin 30^o}{\sin 80^o} \approx 10N.$$
$$T_2 = P.\frac{\sin 70^o}{\sin 80^o} = 2.10.\frac{\sin 70^o}{\sin 80^o} \approx 19N.$$
Teacher: the law of Sines also has many applications in informatics. The following example is an illustration of drawing shapes by the law of Sines on GeoGebra software that helps accurately find the shape of the locus.

Example 7
An angle with two sides always passes through two fixed given points A and B and has a size equal to a given-α, prove that the set of vertices is two arcs.
Teacher: Drawing shapes and finding the set of vertices for this problem is a difficult problem of dynamic geometry software. Here we will use the law of Sines. We draw shapes as follows:
Step 1. Drawing shapes
- Draw angle xIy with the magnitude α.
- Draw a circle-$(O;\, a)$, which a is the length of some line.
- Draw an M-point on (O).
- Draw the OM beam.
- Draw a line d passing through A and parallel to the OM beam.
- Draw Ot ray on d so that Ot has the same direction as OM ray.
- Calculate $\frac{AB}{\sin\alpha}.\sin BAt = m.$
- Draw a circle $(B;\, m)$ to cut Ot ray at a point P, we have $APB = \alpha$.
- Hide unnecessary routes.

Step 2. The locus
- Trace point P, move point M, we obtain the set of points A, which are two symmetrical arcs through AB. (Nguyen, N. G, 2010)



Comment

We have $\frac{AB}{\sin\alpha} = \frac{BP}{\sin BAP}$. *Or* $BP = \frac{AB}{\sin\alpha}.\sin BAP$. *From that, we have a way to draw the shapes as above.*

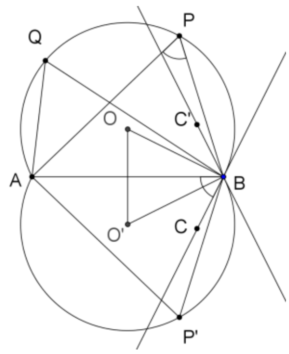
We prove it as follows

The compliance

We will show that the set of vertices is two symmetrical arcs through A, B and remove two points A, B.

Indeed, draw $AQB = \alpha$, so $AQB = ABC$ (or equal to ABC') $= APB$ (or equal to $AP'B$), so the four points A, B, P (or P') and Q are on the same circle (the two triangles share the same bottom and have the equal angles respectively)).

So Q is on APB arc (or $AP'B$ arc), but cut off two points A and B.



The reciprocity

Teacher: Read and prove yourself as homework.

Teacher: Above is an application of the law of Sines in informatics. Come home and find other applications of the law of Sines yourself.

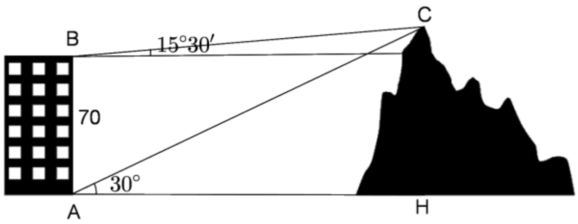
4. Pedagogical experiments

4.1. Experimental purposes

Pedagogical experiments were performed to test the feasibility and effectiveness of the teaching method of Sines function law according to the constructivist theory by us.

4.2. Organization and experimental content
4.2.1. Experimental organization
Pedagogical experiments were performed at Phan Dang Luu High School, Binh Thanh District, Ho Chi Minh City, Vietnam.
+ Experimental class: 10C09.
+ Control class: 10C10
The Experimental period was performed from September to November 2018.
The teacher of the experimental class and control class: Do Thai Phuc.
With the consent of the Board of Directors of Phan Dang Luu High School, we studied the results of the grade 10 classes of the school and found that the general level of Math in both 10C09 and 10C10 classes are equivalent.
On that basis, we propose experiments in class 10C09 and take class 10C10 as a control class.
School administrators, teachers of Mathematics team and 10C09 and 10C10 teachers accepted this proposal and facilitated us to conduct experiments.

4.2.2. Experimental content
The experiments were performed in the lessons on the scalar product of two vectors. After the experiments, we have students take tests. The content of the tests is as follows:
Test 1 (Time allotted: 15 minutes)
From the two positions A and B of a building, one observes mount C of the mountain (figure).



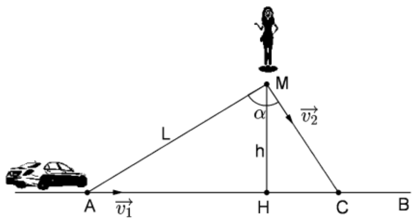
Given that the height AB is $70m$, the AC view intersects with the horizontal direction of angle 30^o, the BC view intersects with the horizontal direction of angle $15^o30'$. How many meters high is the mountain above the ground?
The answer and the scale of the No. 1 test are as follows:

Answer	Scale
From the hypothesis, we infer that the triangle ABC has: $$CAB = 60^o,\ ABC = 105^o30',\ c = 70.$$ $$C = 180^o - (A + B) = 180^o - 165^o30' = 14^o30'.$$	2.5 points
According to the law of Sines, we have: $\dfrac{b}{\sin B} = \dfrac{c}{\sin C}$, or $\dfrac{b}{\sin 105^o30'} = \dfrac{70}{\sin 14^o30'}$. So $AC = b = \dfrac{70\sin 105^o30'}{\sin 14^o30'} \approx 269,4\ (m).$	2.5 points

Assume that CH is the distance from C to the ground. The right triangle ACH has the side CH opposite to the angle 30^o, so $CH = \frac{AC}{2} \approx \frac{269,4}{2} = 134,7\ (m).$	2.5 points
So the mountain is about 135m high.	2.5 points

Test 2 (Time allotted: 20 minutes)

A person was standing at point M, a distance from straight road AB: h = 50 meters to wait for a car. When he saw the car at point A, a distance from him: L = 200 meters, he started running to the road to catch the vehicle as shown.



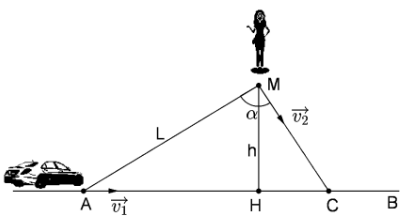
The velocity of the car is $v_1 = 10m/s$. The velocity of the person is v_2. Given that the car and the person move straight and steadily.

a) If $v_2 = 5\ m/s$, which direction must the person run to catch the car (the person comes to the road at the same time or before the vehicle gets there)?

b) In what direction do people have to run, so that v_2 is the smallest? What is the minimum value v_2?

c) If the person wants to meet the car at H (MH perpendicular to AB), what speed of v_2 must that person have?

The answer and the scale of test 2 are as follows:

Answer	**Scale**
a) The person's running direction when $v_2 = 5\ m/s$. Call C the point where the person meets the car; t_1 and t_2 are the movement time of the vehicle and the person respectively. Call α the angle by the direction of the person running with AM.  Applying the Law of Sines to the triangle AMC, we have:	2 points

$\frac{\sin\alpha}{AC} = \frac{\sin\beta}{MC} \Rightarrow \sin\alpha = \frac{AC}{MC}\sin\beta$ with $AC = v_1 t_1$; $MC = v_2 t_2$; $\sin\beta = \frac{h}{L} \Rightarrow \sin\alpha = \frac{h}{L} \cdot \frac{v_1 t_1}{v_2 t_2}$ (1)	
For the person to catch the car (the person arrives at C before or at the same time as the vehicle): $t_2 \le t_1$ (2). From (1) and (2) we infer $\sin\alpha \ge \frac{h}{l} \cdot \frac{v_1}{v_2} = 0,5 \Rightarrow 30^o \le \alpha \le 150^o$.	2 points
b) The person's minimum running speed is From (1), we conclude: $v_2 = \frac{hv_1 t_1}{Lt_2 \sin\alpha}$ (3)	2 points
From (3) we conclude that v_2 is the smallest when t_2 is the largest equal to t_1 and $\sin\alpha$ is the largest equal to 1. Inferred $v_{2_{\min}} = \frac{hv_1}{L} = 2,5\ (m/s)$; then $\alpha = 90^o$, i.e. MC is perpendicular to AM.	2 points
c) The velocity of the person to meet the car at H. We have: $\tan\beta = \frac{MH}{AH} = \frac{v_2 t_2}{v_1 t_1} \Rightarrow \frac{h}{\sqrt{L^2 - h^2}} = \frac{v_2 t_2}{v_1 t_1}$ (4) From (2) and (4), we have: $v_2 \frac{hv_1}{\sqrt{L^2 - h^2}} = 2,58\ (m/s)$. At that time, the person must run in the direction of MH.	2 points

4.3. The evaluation of experimental results

4.3.1. Qualitative evaluation

Before conducting the experiments:

- Students have difficulty in solving practical problems. The ability to associate and connect knowledge is limited.
- Students do not know how to apply the law of Sines in solving physics problems. The ability to integrate mathematics and physics is weak.
- The ability to incorporate mathematics and informatics is limited. Students cannot find applications of the law of Sines in information technology.

After conducting the experiments:

- Students know how to apply the law of Sines in real math problems.
- Students know how to use the law of Sines in solving physics problems.
- Students are excited to find more examples of using the law of Sines in predicting the shape of a set of points of locus problems.

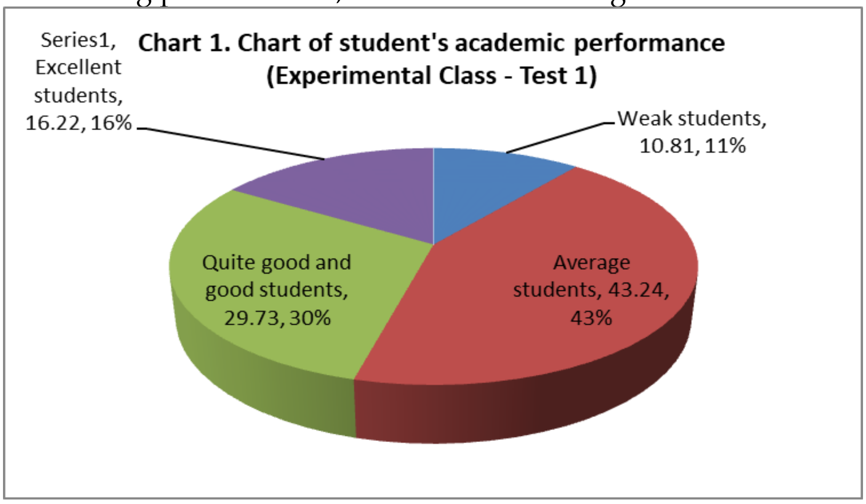
4.3.2. Quantitative evaluation

The test results of experimental and control classes are shown through the following two tables:

Table 1. The results of test 1

Score / Class	3	4	5	6	7	8	9	10	Total
Experimental	1	3	8	8	5	6	4	2	37
Control	1	7	9	8	4	3	2	1	35

The experimental class has 91.89% of the students that score above average, of which 45.95% of the students are quite good (points 7 or above) with two students achieving perfect scores, four students scoring 9.



The control class has 77.14% of the students achieving an average score or above, of which 28.57% of the students achieving a fairly good score (score of 7 or above) 1 student with a perfect score, with two students achieving 9 points.

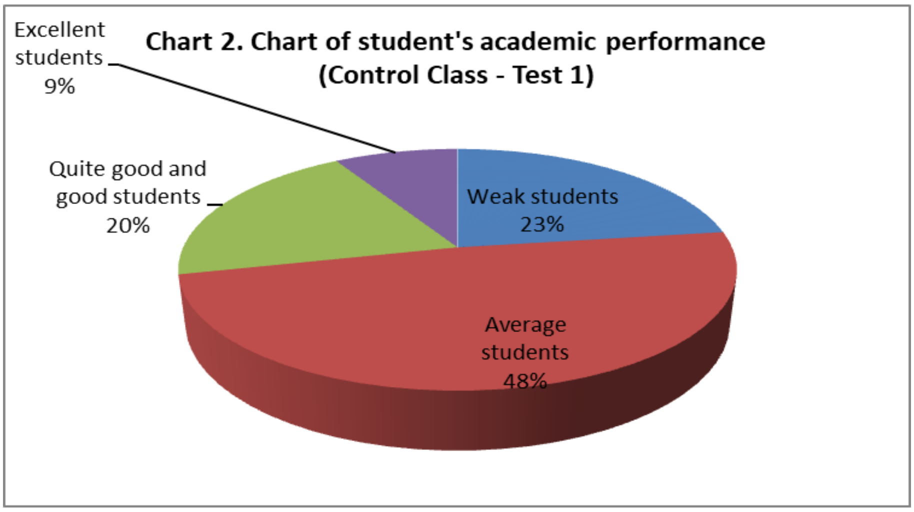
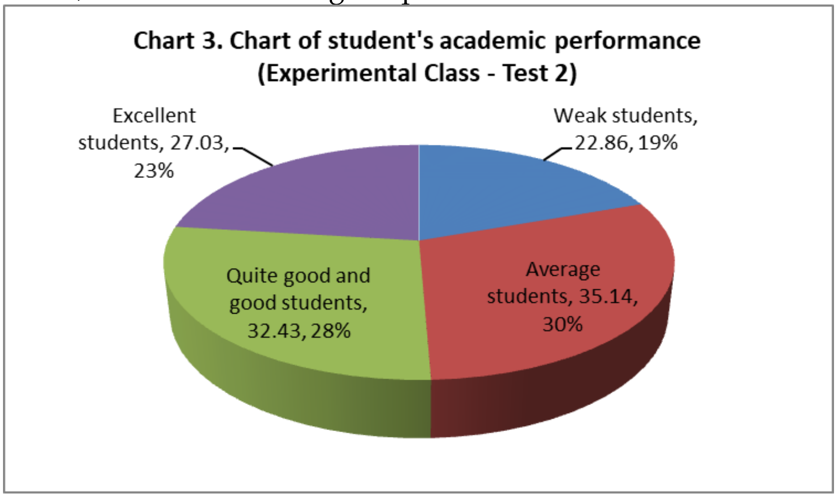


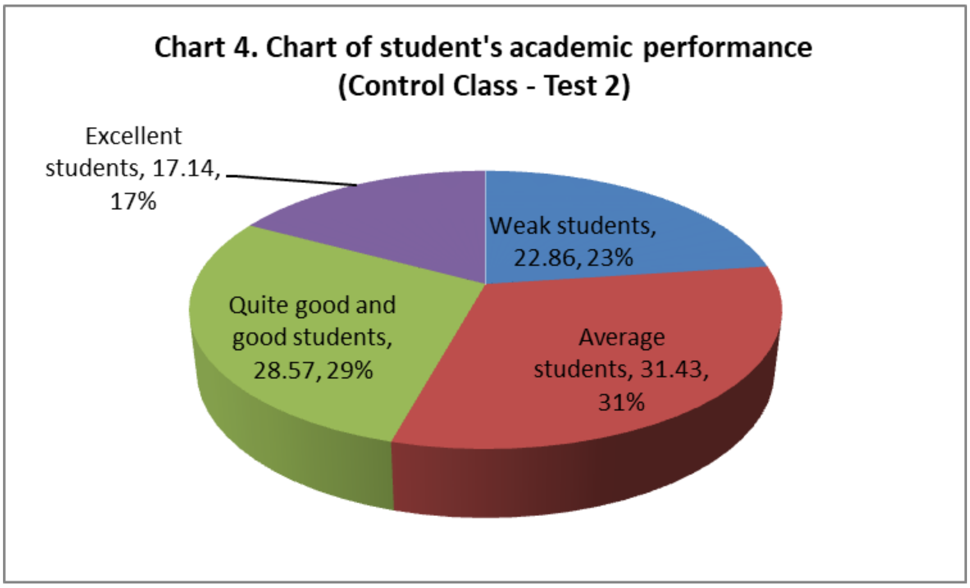
Table 2. The results of test 2

Score / Class	3	4	5	6	7	8	9	10	Total
Experimental	1	1	7	6	5	7	6	4	37
Control	1	7	6	5	5	5	4	2	35

In the experimental class, 94.59% of the students score above average, and 59.46% of the students get good grades (7 or more), four students who get perfect scores, six students who get 9 points.



The control class has 77.14% of the students achieving average scores or higher, of which 45.71% of the students with good grades (7 or more), two students achieving perfect scores, four students with 9 points.



Based on the test results, we can initially see the effectiveness of pedagogical measures to train students the ability to learn the law of Sines according to the constructivist theory that we have proposed and implemented in the experimental process.

4.3.4. General conclusions about the experiments

The experimental process with the results obtained after the tests shows that the experimental purpose has been made, the feasibility and effectiveness of the measures have been confirmed. Implementing teaching methods of the law of Sines in the constructivist theory, contributing to improving the efficiency of teaching Maths for high school students.

5. Conclusion

Teaching the law of Sines according to the constructivist theory, helps promote internal force of learners. Learners themselves take knowledge and deepen knowledge development through two ways of assimilation and adaptation. Adaptation helps learners gain new knowledge and assimilation helps learners expand and develop new knowledge. The constructivist teaching method has many outstanding advantages compared to the traditional teaching method. That is the way to develop the learners' capacity. Learners are the center and main subject of this teaching process. Teachers are just guides through suggested questions to bring students to new knowledge. For the teaching process of constructivist teaching to become vivid and attractive, in the step of designing the situation, the teacher should choose a practical example to make students fall in love with the lesson. The teacher then presents the obstacles that are teaching phases. Each teaching phase has certain difficulties. Teachers absolutely should not assign tasks that are too difficult for students, but rather moderate tasks in the nearest developmental area of each student. If the task is too easy, the student is not interested, but if the task is too high, the student wants to give up. After bringing students to new knowledge, the teaching process there cannot be stopped. Teachers need to dig deep, suggest analytic for them to expand the problem, explore new developments from the initial problem. It is the process of creativity and the highest thinking of all the thinking scales according to the new Bloom rating scale.

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