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Enhancing Upper Secondary Learners' Problem-solving Abilities using Problem-based Learning in Mathematics

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Abstract. Developing problem-solving abilities is a major objective of learning mathematics at school. However, learners' problem-solving abilities are still critical. The main purpose of this study was to investigate how the problem-based learning model could enhance learners' problem-solving abilities in mathematics. The study used quasi-experimental research with one group pre-test-post-test design. The population in this study consisted of fifty-four grade eleven learners (aged between 16 to 19 years old) from one school in Kayonza District in Rwanda. Data were collected using mathematical problem-solving tests and interviews and were analysed using paired t-tests for dependent sample means and descriptive analysis. The study results indicate that problem-based learning potentially impacts learners' problem-solving abilities. It is shown from learners' work in problem-solving that all indicators of problem-solving abilities, namely understanding the problem, planning ways to approach the problem, monitoring the progress while tackling the problem and reviewing the solution process, emerged as being fairly well improved. In addition, based on the interview results from some learners and their teachers, they like the PBL model because embedded tasks helped them to apply the knowledge that can improve their reasoning, creativity and thinking capability. The study recommends that schools encourage teachers to adopt PBL for enhancing learners' problem-solving abilities. Additionally, researchers are urged to use findings from this study as a reference for further research. Furthermore, researchers could conduct similar research on a large scale using different methodologies.

Keywords: mathematical problem-solving; problem-solving abilities; problem-solving processes; problem-based learning; secondary school learners; Rwandan schools

1. Introduction

The teaching of mathematics for problem-solving has been central to mathematics education since the movement of curricula reform around the 1980s in the United States and has expanded worldwide (Rosli et al., 2013). The ultimate goal is to engage learners in conducting mathematical investigations by themselves and for them to be able to identify the application of mathematics they have learned in a real-world situation (Stacey, 2005). The introduction of Rwanda's competence-based curriculum (CBC) was meant to address this educational reform (REB, 2015). However, learners' problem-solving abilities still require attention. Research of international assessments such as the Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMSS) shows that the levels of learners' problem-solving abilities are still weak, even in developed countries (Novita et al., 2012; Stacey, 2005). Therefore, the situation in developing countries such as Rwanda can only be imagined. Factors such as mathematics learning and the evaluation process contribute to low levels of problem-solving. Thus, it is essential to update the learning and evaluation process to ensure quality education is oriented to problem-solving.

Problem-solving refers to mathematical tasks that have the potential to provide intellectual challenges for enhancing learners' mathematical development (Novita et al., 2012). These tasks are understood by learners but cannot be solved by an already established procedure because they have a hurdle that is not immediately realised. Learners must then think and reason through a situation. It is good to note that what seems a problem to someone is not necessarily a problem to others: it depends on the situation and the problem solver (Schoenfeld, 1992). The solving of problems can foster the development of 21st-century skills essential for the workplace (Gravemeijer et al., 2017; Szabo et al., 2020) and stimulate the interest, motivation and curiosity of learners (Căprioară, 2015). In addition, having to solve an appropriate problem can affect the enhancement of learners' problem-solving abilities in identifying how they can use mathematics skills to solve daily problems they face in life. It is essential to expose learners to meaningful problems to practise sense making (Gravemeijer et al., 2017; Polya, 1945; Voskoglou, 2011; Yu et al., 2014).

Problem-solving abilities are fundamentals of mathematics education in schools. The enhancement of problem-solving abilities can equip learners to think logically, analytically, critically, and creatively (Surya et al., 2017). Therefore, learners' problem-solving abilities will be enhanced if real-life problems are used in the classroom. Yu et al. (2014) contended that problem-solving abilities are better cultivated when concrete problems are taught in the classroom. Klegeris and Hurren (2011) further pointed out that skills necessary to solve a problem from available knowledge and information are developed while critically analysing the contextual problems in a small collaborative group. Moreover, Setiawan and Supiandi (2018) pointed out that successful problem-solving abilities are emphasized when learners develop their own knowledge and apply concepts. In this regard, this study used problem-based learning (PBL) as a learning model because it offers many opportunities for learners to work on solving concrete problems during learning. It has been used in an effort to help learners improve their problem-solving abilities in mathematics (Prastiti et al., 2020; Setiawan & Supiandi, 2018).

PBL is a learner-centred pedagogical model that offers a learning environment with a problem-solving focus (Klegeris & Hurren, 2011). The general pattern of instruction with PBL is to present concrete real-life problem (related to the content and the broader curricular goals) to learners at the start of the lesson. Learners are then given time to work collaboratively in small groups to identify what they need to know to solve the problem while engaging in self-directed learning (SDL) to look for answers and report back to the group and apply the new knowledge to the problem. Finally, groups present their proposed solution to the problem and conclude the activity by reflecting on what they have learned as well as the effectiveness of the strategies employed. The entire process of learning within PBL occurs around solving problems (Savery, 2019). Thus, PBL is used in this study to provide learners with plenty of opportunities to solve problems during learning.

Some studies documented the effectiveness of PBL model in different contexts (Astriani et al., 2017; Merritt et al., 2017; Savery, 2019; Siregar, 2017; Valdez & Bungihan, 2019). They have shown that PBL promotes conceptual understanding and improves higher order thinking skills, confidence, knowledge retention, motivation, academic achievement, and problem-solving. Klegeris and Hurren (2011) point out that learners develop transferable skills while analysing the problem, constructing their knowledge and applying concepts in PBL (Setiawan & Supiandi, 2018). However, other studies have shown that PBL can lead to poor performance when it is practised in a fact-based learning environment (Aksela & Haatainen, 2019; Craig & Marshall, 2019). PBL can also disengage learners unfamiliar with problems from participating in group activities.

Parallel to this, some studies have employed different learning strategies and focused on key factors to enhance grades five through to eight learners' problem-solving abilities in mathematics (Bostic, 2011; Sigurdson et al., 1994; Verschaffel et al., 1999; Yu et al., 2014). These studies implemented teaching strategies that supplemented daily mathematics instruction with a problem-solving focus for at least four weeks. The findings of these studies revealed enhanced learners' problem-solving abilities in terms of understanding the problem, implementing the plan for solution and verification of the answer, as well as improved achievement and attitudes. Several other studies (Darma, 2018; Klegeris & Hurren, 2011; Setiawan & Supiandi, 2018) exposed learners to learning mathematics using PBL to improve problem-solving abilities. The common outcome of their studies is a positive impact of PBL on learners' problem-solving abilities. The latter show evidence of the suitability of PBL in mathematics for small and large classroom settings. However, to the authors' knowledge, few studies have applied PBL in upper secondary schools to enhance learners' problem-solving abilities. Additionally, no study has used PBL to strengthen problem-solving abilities in Rwanda. Thus, the current research will document literature in this aspect and add knowledge to the existing body of knowledge.

The reviewed literature revealed that the current learning and the evaluation process of mathematics hinders the provision of quality mathematics education. In addition, learning contributes nothing to learners' problem-solving abilities when tasks are meaningless. Consequently, learners will not be able to solve problems successfully in mathematics or beyond and will lack the significance of education in general, particularly mathematics. However, solving problems is the primary means of doing mathematics at school (Stacey, 2005). Thus, this study intends to investigate whether problem-based learning can enhance the problem-solving abilities of grade eleven learners in Kayonza District in Rwanda. The following research questions guided the study:

1. Will problem-based learning models improve learners' problem-solving abilities?
2. What do teachers and learners experience while learning mathematics with PBL model?

Theoretical approach

According to Polya (1945), problem-solving is an attempt to find a way out of the difficulty in achieving a goal which is not immediately achievable. The entire process of problem-solving comprises identifying and understanding the problem, planning ways to approach the problem, monitoring the progress while tackling the problem and then reviewing the solution to the problem (Novita et al., 2012). Polya (1945) suggested four stages for problem-solving: (1) understanding the problem (what is known and needs to be done, and then deciding what information is important and what seems unimportant); (2) devising a plan (reflecting on ideas that can be brought to the problem); (3) carrying out the plan (following through with the approach selected, carefully taking each step along the way) and looking back (verifying whether the solution makes sense or fits the given data). These processes are used in this study as indicators of problem-solving abilities.

The PBL strategy in the classroom reflects the constructivism learning theory, specifically Vygotsky's social constructivism theory (Vygotsky & Cole, 2018). This theory states that knowledge is constructed and reconstructed socially between teachers and learners where learners learn from one another in small groups (Vygotsky, 1978). In this learning, the teacher acts as a facilitator or a guide and catalyst. Learners actively engage in the learning process while making interpretations and constructing new knowledge through teamwork and collaboration (Cohen et al., 2003). Social interaction support learners to learn concepts more quickly and efficiently than when working alone. This interaction creates a better opportunity for learners to build their problem-solving abilities (Vygotsky & Cole, 2018).

In the PBL, learning opportunities are offered to learners at the beginning of the lesson because learners are not empty-minded (Razieh, 2016). Learning is maximized in the so-called zone of proximal development (ZPD). According to Vygotsky (1978), the ZPD refers to the distance between the actual and the potential development of an individual. This is why the ZPD could not be ignored in this study. Teachers should monitor individual learners to promote discussions that support interactive learning or conceptual understanding. PBL emphasizes collaborating and sharing of information, which positively impact learners' problem-solving abilities.

2. Methodology

This study is a mixed-method research of quasi-experiment with one group pretest-posttest design. Qualitative data were gathered from interviews while quantitative data were obtained from mathematical problem-solving tests. The choice of mixed methods was guided by the belief of pragmatic philosophers that mono-paradigm orientation research is not enough to interpret and understand human behaviours (Alise & Teddlie, 2010). More practical and pluralist approaches should allow a combination of research methods regarded as most appropriate to address the research questions (Kivunja & Kuyini, 2017). The epistemological and ontological views of the research problem support mixed research methods. Thus, for this study, mixed methods were considered appropriate.

Using non-probability purposive sampling techniques, eighty-two (82) grade-eleven learners (45 females and 37 males) aged between 16 to 19 years old from one school in Kayonza District in Rwanda were selected. The school accommodates both core and subsidiary mathematics in the combinations and is situated in the district in which the University of Rwanda – College of Education (UR-CE) operates. These learners have one year left to complete their secondary education and were divided into two combinations; forty-seven in Mathematics-Economics-Geography (MEG) and thirty-five in Physics-Chemistry-Biology (PCB). These learners received a PBL intervention for 14 weeks. Therefore, their teachers could not be excluded from the study. They were given a test before and after the intervention. However, the study considered only fifty-four learners (28 females and 26 males) who completed both pre-test and post-test.

Tests comprised seven questions and were selected from a bunch of questions prepared for a large doctoral project by the first author based on their fit for enhancing problem-solving abilities as this study intended. Researchers formulated questions based on the algebra content of grade eleven learners (see students' book on REB website) on units of sequences, logarithmic and exponential equations, and solving equations and puzzle problems. To avoid bias, questions from the pre-test were slightly modified or replaced in the post-test (see Table 1).

Table 1: Pre-and post-test mathematics items arrangement

Units	Question number for pre-test	Question number for post-test	Comments
Unit 2: Sequences	1,2	1,5	The two questions were similar except that they were arranged in different positions.
Unit 3: Logarithm and exponential equations	7	7	These two questions were the same.
Unit 4: Solving equations	6	2,3	Question 3 in the post-test covered the unit of solving equations using concepts of matrices. Q6 and Q2 were different.
Puzzle problems	3,4,5	4,6	Puzzle problems for the pre-test were completely dissimilar to those of the post-test.

Two experts, both PhD holders in mathematics education, validated the tests before their administration. They rated the question items as very easy, easy, and difficult. In addition, they provided comments on whether they fit research objectives, and these were used to revise the questions. Additionally, a reliability coefficient was calculated using a test-retest method. It was found to be .72 from a sample of 30 learners who were not part of the study in the actual data collection.

This research received an ethical clearance letter from UR-CE through the research and innovation unit. This letter was used to seek permission to collect data from the district education office where the school is located. Before commencing the data collection process, participants were explained the purpose of the study to encourage their participation in the research. They were given time to ask questions and fill out consent forms to ensure voluntary participation. After that, a pre-test was given to monitor learners' level of problem-solving abilities prior to the intervention.

Five mathematics teachers (1 female and 4 males) at the school of intervention attended a two-day workshop on PBL. The workshop focused on PBL concepts, patterns of instruction within PBL and the formulation of PBL activities. At the end of the workshop, teachers prepared and performed two microteachings using PBL and reflected on their teaching in order to make better use of PBL. Two out of five teachers implemented PBL for 14 weeks and were assisted in refining the intervention during the intervention period. Teachers encouraged learners to identify and understand the problem presented to them, to plan ways to approach it, to monitor the progress while tackling the problem, and to review the solution. Learners learnt by doing problems before receiving input from the teachers (see introduction section for more detail).

Box 1. Example of activities done as a teaching intervention

Activity:

A father wants to build a tower of ten rows of blocks starting from the bottom. Each row will have two blocks fewer than the previous. He would like to know the total number of blocks he will need to purchase before starting with the bottom row comprising 1025 blocks. What is the total number of blocks he will need?

Steps followed:
The problem was presented to learners at the start of the lesson (the teacher wrote the problem on the chalkboard) (5 minutes). They were then given time to read the problem and brainstorm ideas critically, and difficult words were explained (10 minutes). The teacher assigned learners in small groups to analyse the problem (2 minutes). During group work, learners did research to identify what they needed to know to solve the problem (using textbooks), propose solutions (20 minutes), and then report back to the group to apply the new knowledge to the problem. The entire process of problem-solving (1-understand, 2-plan, 3- implement, and 4-check the solution) was written on the chalkboard in one corner so that even those who might have forgotten the steps could verify them. Learners were given time to present their findings to the whole class, where reflections were given by classmates (15 minutes) (group presentation was given to those who solved the problem in different ways; not all the groups in the class). A decision was taken for the whole class, and the teacher linked the problem solution with the lesson's learning objective. The remaining time was given to learners to foster the concepts by working on various textbook exercises which the teacher indicated in their book.

Note:
*Many of the problems used during the intervention were designed based on the content, and textbooks were used as well. Questions were used for sequences (the lesson was designed for 80 minutes).
*The lesson's learning objective was to be able to apply a formula to determine the n^{th} term of arithmetic sequences and find the sum of n in terms of arithmetic progressions.
*Learners had no idea about the formula for finding the sum of n^{th} arithmetic progressions or even how to find any term of the sequence; it was a new lesson. The formulas were clarified (the general formula was given in case learners did not find that formula).

Every Friday, teachers in the intervention and researchers met to evaluate the intervention's progress, discuss difficulties, and prepare the next lesson by focusing mostly on the PBL activity. To recap, researchers made suggestions and contributed to lesson preparation at the end of the meeting. These meetings were organized and conducted for mentorship purposes. At the end of each week, researchers conducted semi-structured interviews with teachers and FGDs with selected learners to hear their appreciation of the intervention. Field notes were also taken and were used to refine the learning material and improve teacher questioning strategies. After intervention, a post-test was administered to the same group of learners to monitor their knowledge and skills gained in problem-solving and any change in their problem-solving abilities. Researchers supported learners to understand the language used in both pre-test and post-test. Each test was conducted for approximately 60 minutes.

Before data analysis, every question on the answer sheet of learners was checked and marked. Each question was allocated ten marks following indicators of problem-solving abilities (see details in Table 2), as Szetala and Nicol (1992) suggested. The ability to understand the problem scored 0 to 2, the ability to plan the solution strategy scored 0 to 2, the ability to implement the plan was scored from 0 to 4, and the ability to check whether the solution makes sense scored 0 to 2.

Table 2: Details of the indicators and scoring of problem-solving abilities

	Indicators	Details	Scores
1	Understanding the problem	If there is no identification of known, asked elements of the data needed and underlining of keywords in the question	0
		If there is the identification of known, asked elements, or underlining of keywords but are incomplete	1
		If there is the identification of known, asked elements, or underlining of keywords and are complete and adequate	2
2	Planning the solution strategy	There is no plan at all, or it is unclear, or it is erased	0
		There is a plan with errors	1
		There is a correct plan	2
3	Implementing the plan	There is no answer, or it is erased	0
		Some solutions are following the plan but are wrong	1
		Some solutions are following the plan with errors and are incomplete	2
		There are correct solutions that follow the plan but are incomplete	3
		There are correct solutions that follow the plan	4
4	Interpreting or checking the solution obtained	There is no conclusion or interpretation of the solution obtained	0
		There is proof of the solution, or there is interpretation with errors	1
		There is correct proof of the solution obtained and interpretations	2

Microsoft Excel was used as a tool to analyse data. Data from interviews were analysed using descriptive analysis (presented in text and quotes) while data from tests were analysed using charts and inferential statistics of the paired t-test. The test was computed to determine whether learning mathematics with PBL has an impact on learners' problem-solving abilities. The authors wanted to find out whether there is a significant difference between means of learners' scores in the pre-test and post-test at a .05 level of significance. Finally, normalized learning gains were computed from pre-test and post-test results across each indicator of problem-solving abilities.

3. Findings

The results of the study are presented based on the order of the research questions examined.

Question one

To establish how learning mathematics with PBL improved learners' problem-solving abilities, data from the pre-test and post-test results for each question were analysed according to indicators of problem-solving abilities described in Table 2. Learners' problem-solving performance on each of seven questions according to problem-solving indicators is displayed in Figure 1. Results indicate an average improvement of learners' problem-solving abilities from 37% to 61%.

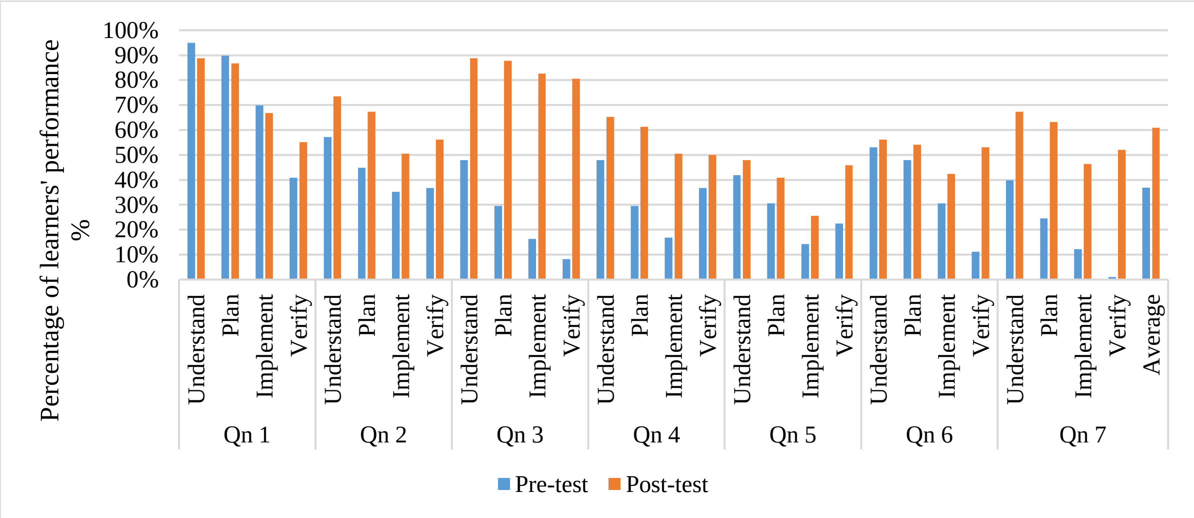

Figure 1: Percentage of learners' problem-solving performance on each question according to indicators of problem-solving abilities (Qn: Question)

The performance percentage on each question according to problem-solving indicators reveals that learners performed better in the post-test than in the pre-test. This means that there is an improvement in learners' problem-solving abilities. Also, a paired t-test for the dependent sample mean was computed (see Table 3) to explore whether there is a significant difference between the means of learners' scores in the pre-test and post-test. The results revealed $t\ stat = -12.08 < t\ critical\ two\ tail = 2.01$, which means that PBL instruction affects learners' problem-solving abilities positively, and the impact is significant with $p < .05$.

Table 3: t-Test: Paired two samples for means

	Mean	Variance	N	Pearson Correlation	df	t Stat	P(T<=t) two-tail	t Critical two-tail
Pre-test	22.11	37.50	54	0.27	53	-12.08	0.00	2.01
Post-test	37.30	76.85	54					

Based on the results displayed in Figure 1, there is little impact on learners' abilities for question one after the intervention. The figure shows that the first three indicators (understand, plan and implement) on question one are higher in the pre-test compared to those of the post-test. This could be because the question was too short, with few words embedded. Also, learners could have been accustomed to the type of questions which seem closed with few terms that generally provide an immediate procedure to apply for solving the problem. An example of a learner response in Figure 2 shows the application of looking for a pattern strategy to find a solution in the pre-test. However, in the post-test, the learner could apply more than one strategy for an answer, although s/he was not successful.

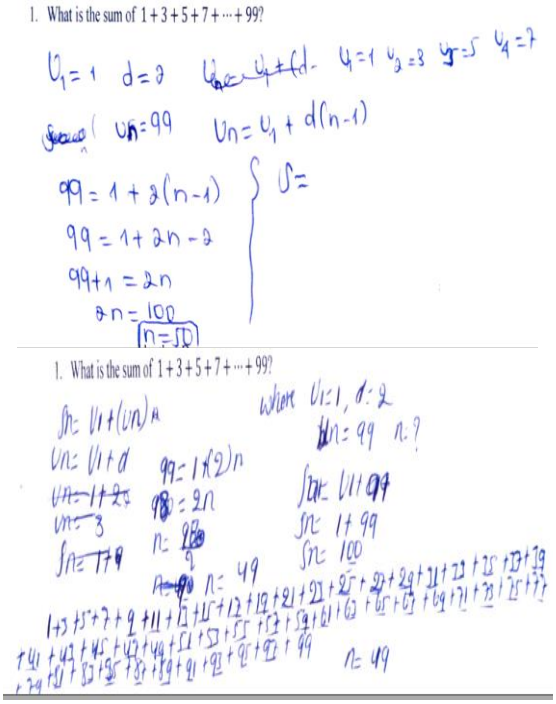

Figure 2: A learner response on question one in the pre-test and post-test

Based on responses shown in Figure 2, a learner made better progress in the post-test than in the pre-test of problem-solving ability. The first attempt shows that the learner knew the formula and relied on it to find the solution. However, for the second attempt, the learner answered the question freely, showed all the workings, and performed better than on way to the solution.

The effect of PBL on learners' problem-solving abilities was evaluated according to gender by minimum, maximum, quartiles, median, mean, range, interquartile range, *p*-value, and learning gains (see Table 4).

Table 4: Effect of problem-based learning on learners' mathematical problem-solving abilities

	Pre-test [All, N=54]	Post-test [All, N=54]	Pre-test [Female, N=26]	Post-test [Female, N=24]	Pre-test [Male, N=28]	Post-test [Male, N=28]
Minimum	14%	30%	19%	30%	14%	31%

Quartile 1	24%	43%	25%	43%	25%	46%
Median	31%	51%	30%	49%	33%	59%
Quartile 3	37%	61%	38%	59%	39%	68%
Maximum	49%	77%	49%	74%	49%	77%
Mean	31%	53%	32%	51%	32%	56%
Range	34%	47%	30%	44%	35%	46%
Intra Quartile Range	13%	19%	13%	16%	15%	22%
p-value	<.001		<.001		<.001	
Learning gains	32%		28%		36%	

The learning gain (N-gain) for males is 36%, while that of females is 28%. The results indicate that the overall learning N-gain is 32%; however, male learners outperformed female' learners in both pre-test and post-test. Table 4 presents the detailed results.

Furthermore, the average N-gains across each indicator of problem-solving abilities were computed. The results indicate an overall N-gain of 38%, which shows to what extent changes occurred in learners' problem-solving abilities. Results show a higher N-gain of 43% of the "verify" indicator. This means that after PBL, learners learn to make sense of the solution to the problem. However, it was not easy to identify a high N-gain for the "implement" stage because learners mostly combine the "plan" and "implement" phase when solving the problem. Thus, effort should be made to indicate the implementing phase during the problem-solving process. Figure 3 shows the graph of indicators of problem-solving abilities against percentages (values).

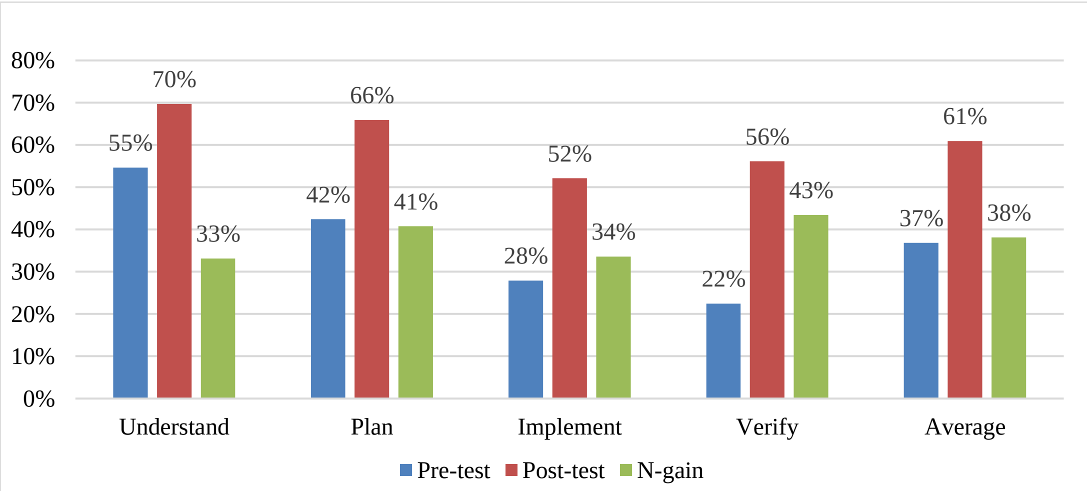

Figure 3: Average N-gain across all questions for each indicator of problem-solving abilities

Moreover, the study compared indicators of learners' problem-solving abilities based on the question content, sequence, and series, solving questions using numerical methods, logarithm and exponential equations, and puzzle problems, as shown in Figure 3. The results indicates an improvement of learners' problem-solving abilities in solving equations, puzzle problems, logarithms and exponential equations and a slight decrease of learners' problem-solving abilities in sequence and series.

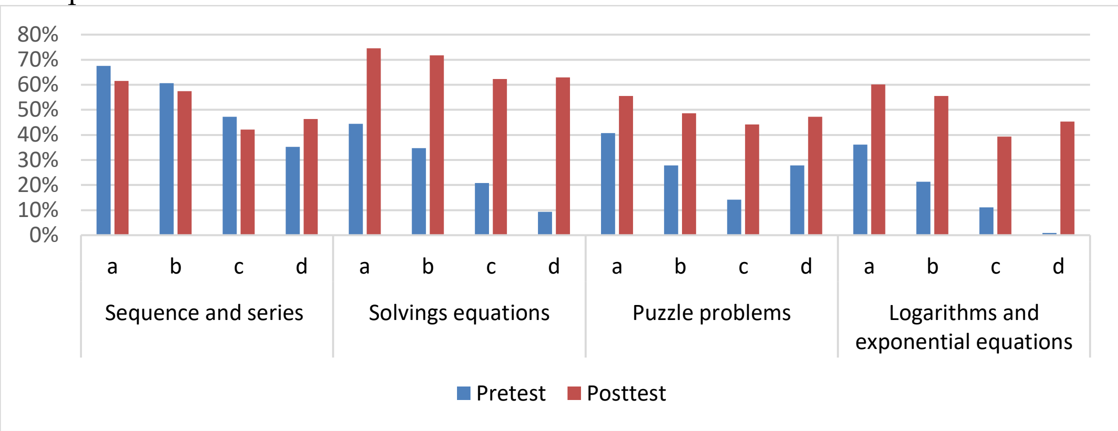

Figure 4: Learners' problems-solving abilities according to the content

Question two

Results from the interview data indicate some PBL benefits that support the enhancement of learners' problem-solving abilities. The first finding relates to the PBL environment which provides a learning atmosphere that can help learners to develop their understanding. Both teachers and learners indicated that they were able to value the importance of understanding a problem before they can solve it. The PBL activity requires learners to reason through the situation. An example of a transcript from Anna is given below:

> *"Usually, the teacher gives us notes before learning, and we get tired before learning new knowledge, but with this learning style, we learn more meaningfully with understanding."*

Anna continues:

> *"Normally when you are given a challenging activity before learning, it first challenges you before getting the right answer…it requires us to use our brain as compared to how we usually do… normally we memorize the formula so that if they change the question a little bit, you cannot think deeply, but now we are required to think first so that once you face the similar challenge, you are not afraid to solve it".*

This finding indicates that PBL learning assisted learners in developing mathematical understanding. PBL learning offered a space for the learners to learn easily as they had enough time to reason and think without getting tired or bored. The second finding relates to PBL activity which can support learners in planning ways of approaching the challenge since the activity did not require them to use any readymade procedure. Many learners were of the view that the PBL strategy made the learning of mathematics more concrete and enjoyable than being abstract, where formula and rote learning are given less attention. Frank described this as follows:

"Activities are severe and good because they reflect the real-life situation that we are familiar with and are not forcing us to use the formula from the notebook, but the brain and different strategies".

Many learners indicated that they had received different guidance from their teachers, which again encouraged them to think critically. This means that both teachers and learners could pay attention to processes; teachers could not expect short or final answers from the learners. Rather, they could shift their teaching to focus on problem-solving. The usual teaching process was as follows:

"Teachers often give us exercises and homework sometimes and leave them uncorrected… consequently, engaged learners cannot know if what they have done is correct or know the wrong step; those unengaged learners get nothing".

The results indicated that being given challenging activities helped learners to be engaged and reduced the teacher's talking in the classroom. For instance, Hawah indicated:

"I wish to learn in this way…. to learn first before taking notes and therefore we could take notes of what we understand well".

Anna was referring to being given an engaging activity to think about and identify the learning gap. The teacher should intervene to fill the knowledge gap. Fausto complimented Hawah, saying the following:

"Actually, we saw more content of what we are learning here in senior five (grade eleven); what we need most is little guidance from the teacher depending on what he needs us to achieve."

Fausto continued: *"Normally, when one learner takes the role of the teacher, or when we work in small groups, we are not afraid to ask questions than we can do to the teacher*."

The third finding relates to difficulties of learning and implementing PBL. Generally, participants reported positive experiences that are similar to previous studies such as the improvement of learners' attitudes, longer retention of knowledge, improved self-directed learning practices, enhanced problem-solving abilities and critical thinking skills. They also developed a teamwork spirit as well as their presentation and communication skills. However, participants reported negative experiences of PBL, namely that it is time consuming, requires time, and needs infrastructure and equipment, as well as the Internet. Therefore, the PBL learning process could impact on teachers' workload. Below is a transcript of a teacher interview. Kalisa said the following:

"You see, you need to understand the content very well to design appropriate task… sometimes you need to search on the internet, and I have three classes to teach…. Besides, I cannot teach very fast the content I have to cover in 9 weeks of this term".

Kalisa added that:

"*The other is that when you give learners much time to talk in the classroom, they shout out. It becomes difficult to keep one conversation in the room."* He further said that: *"you find yourself spending much* *time managing the classroom instead of managing the 40 minutes of the lesson period."*

Kalisa was complemented by his colleague Kamana, who said:

"PBL methodology is good to the extent I saw learners going beyond the planned learning expectations…but they may not achieve the learning objective, then you delay covering the content as planned on the scheme of work."

4. Discussion

The results of the mathematical problem-solving test revealed that scores were higher in the post-test than pre-test, with average performance increasing from 37% to 61%, which confirms the value of the t-test. This result demonstrates that PBL instruction was effective in assisting learners in improving their problem-solving abilities. Across all questions, learners showed a significant increase in how they demonstrate understanding of the problem, plan ways to approach the problem, monitor their progress as they tackle the problem and review the solution to check whether all conditions of the problem had been met. These findings are in line with those of Padmavathy and Mareesh's (2013) study which found positive results in learners' understanding and abilities to use concepts in real life after learning with PBL. Sigurdson et al. (1994), Verschaffel et al. (1999) and Yu et al. (2014) pointed out that a problem-solving focus should supplement daily mathematics teaching to improve learners' achievement, attitudes and problem-solving abilities.

In addition, the improvement of problem-solving abilities using PBL models follows the social constructivism theory of learning (Purwoko et al., 2019). As Vygotsky (1978) stated, learning is central to students, and new knowledge is constructed owing to social interactions. Students engaged in and sharing the activity are facilitated by their teacher through appropriate tasks and monitoring productive discussions. Thus, students' experience when solving problems in small groups during learning improves their thinking and mental function.

The PBL model effectively cultivates students' abilities to understand and analyse the problem then select and develop solutions. This is why other studies (e.g., Mukuka et al., 2020; Ukobizaba et al., 2021) have emphasized the need for teachers to turn their attention to instructions oriented to problem solving and assessment in order to address learners' learning needs. The study revealed better results of learning, namely N-gain of 32% of the participants. This finding shows that learners benefitted from the PBL method, especially the girls. Boys (N-gain =28%) outperformed girls (N-gain=36%) for both pre-test and post-tests. This finding is in line with the study conducted by Siregar (2017) who found a higher increase in the problem-solving ability of male learners in both control and experimental group. This could be because mathematics is perceived as a male subject and difficult (Osman & Kriek, 2021). However, both male and female need problem-solving experience in mathematics that encourages them to think differently since the labour market is open equally to both.

Changes in learners' problem-solving abilities were high (N-gain=38%). Surya et al. (2017) point out that high N-gain boosts learners' confidence, interest and motivation. A higher N-gain was on the "verify" indicator with 43% compared to other indicators. This means that after PBL, learners are able to make sense of the solution to the problem. It helped them to value the importance of the process in problem-solving. Learners showed their understanding rather than directly solving the problem; they showed their plan on the answer sheet rather than using another sheet as well as showing whether all the conditions to the problem had been satisfied. This indicates how learning mathematics with a problem-solving focus should take place, with minimal attention and focus being given to short and final answers (Novita et al., 2012). The situation then calls for teachers to pay attention to problem-solving processes to help learners to be creative. NCTM (2010) points out that exposing learners to meaningful problems fosters understanding and mathematical development.

To develop the learners' problem-solving abilities in challenging tasks, guidance in feedback on the worked examples and scaffolding were of primary importance (Setiana et al., 2021). As in the zone of proximal development (Vygotsky & Cole, 2018), this helped learners be engaged, reduced teacher talk in the classroom, and enhanced learning and cognitive development. The study conducted by Simamora and Saragih (2019) found that learning using culture context materials improved learners' problem-solving ability and self-efficacy. In addition, according to Szetala and Nicol (1992), paying attention to the process and explanations of learners' solutions rather than focussing on the answers learners give will be more beneficial to learners' communicative thinking. The PBL model accommodates these facilitations. During the problem-solving activity, the teacher visited learners in their small groups to guide the problem-solving process as an example of facilitation. In addition, learners presented findings to communicate their thinking on every step taken to other learners.

Through problem-solving, learners can improve their reasoning. An improvement in N-gain influences learners' abilities, confidence, interest and motivation to learn mathematics (Surya et al., 2017). In other words, meaningful mathematical tasks can equip learners to think logically, analytically, systematically, critically and creatively. Results from interviews indicate participants have positively experienced PBL strategies that are in line with previous studies (Osman & Kriek, 2021; Setiawan & Supiandi, 2018). The PBL strategy made the learning of mathematics more learner-centred and active while providing learners with the opportunity to develop interpersonal relationship skills, critical thinking and reasoning skills, improved attitudes, and enhanced self-directed learning. Also, PBL provided learners with the opportunity to associate academic life with their everyday experience, which substantively increased their awareness of the usefulness of mathematics.

5. Conclusion

Based on the findings and discussions, the study concludes that problem-based learning impacts learners' problem-solving abilities positively in terms of understanding the problem, planning ways to approach the problem, monitoring the progress while tackling the problem and reviewing the solution process in order to check whether all conditions of the problem have been satisfied. In addition, the PBL learning activities provided learners with opportunities to apply knowledge. Furthermore, although PBL presents a negative experience, it nevertheless also has many positive experiences. Thus, teachers should apply the PBL model to prepare learners better for the future and develop their problem-solving abilities. The focus is on how learners approach the problem at the initial stage. The study invites further research to explore the effectiveness of PBL on learners' problem-solving abilities using a different methodology on large scale.

Limitations of the study

The study was limited to two classes of grade eleven learners from one school in Kayonza District in Rwanda. It has used one group pre-test and post-test design. Therefore, there could be other factors that could have influenced the increase in learners' problem-solving abilities. This could have been avoided if an experimental and control group had been used.

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